Maharaja's College Ernakulam

A GOVERNMENT AUTONOMOUS COLLEGE

Re-Accredited by NAAC with 'A Grade'
Affiliated to Mahatma Gandhi University
Centre of Excellence under Govt. of Kerala
Identified by UGC as College with Potential for Excellence

POST GRADUATE AND RESEARCH

DEPARTMENT OF MATHEMATICS

Estd. 1875

Post Graduate Curriculum and Syllabus

(Credit Semester System)

M.Sc. MATHEMATICS

For 2022 Admission Onwards

MAHARAJA'S COLLEGE, ERNAKULAM

(A Govt. Autonomous College)

BOARD OF STUDIES IN MATHEMATICS (PG)

CURRICULUM

FOR

M. Sc. MATHEMATICS PROGRAMME
Programme Code : MRMATPG

UNDER

CREDIT SEMESTER SYSTEM

(MC-PGP-CSS)

(Revised Autonomy Syllabus 2022 onwards)

PREFACE

The present time is experiencing unprecedented progress in the field of Science and technology in which mathematics is playing a vital role; and so the curriculum and syllabi of any academic programme has to be systematically subjected to thorough revision so as to make them more relevant and significant.

Maharaja's college, Ernakulam is a unique institution of higher learning in the state. Its hoary tradition and consistent achievement in various fields of human activity envelop it with a halo of an outstanding temple of knowledge.

The college was elevated to the status of autonomous College by the Government of Kerala and UGC in the year 2014. This is the only government college in Kerala which has been granted autonomy.

The College is also committed to prepare a comprehensive plan of action for Credit and semester system in Post Graduate programmes. Various workshops with the participation of the teachers from affiliated colleges and invited experts from other Universities were conducted at our institution. The syllabus and curriculum we present here is the follow-up of such workshops.

We gratefully acknowledge the assistance and guidance received from the academic and governing council of our college and all those who have contributed in different ways in this venture.

It is recommended that the content of this syllabus be reviewed and adapted in the light of the consultative process and based on its application in future curriculum revision initiatives. The syllabus and curriculum also be revised periodically.

I hope this restructured syllabus and curriculum would enrich the students.

Dr. Bloomy Joseph
Chairman Board of Studies (PG)

MASTER DEGREE PROGRAMME IN MATHEMATICS
BOARD OF STUDIES MEMBERS
Mathematics (PG) - 2021
MAHARAJA'S COLLEGE, ERNAKULAM
(A Govt. Autonomous College)

Sl. No.	Category	Name	Designation
1	Internal	Dr. Bloomy Joseph(Chair person)	Associate Professor
2	Internal	Dr. Jaya S	Associate Professor
3	Internal	Ms. Jaya Augustine	Assistant Professor
4	Internal	Dr. Prakash G N	Associate Professor
5	Internal	Ms. Thasneem T.R.	Assistant Professor
6	Internal	Ms. Jis Mary Jose	Assistant Professor
7	Internal	Ms. Sreeja K U	Assistant Professor
8	Internal	Mr. Sreeraj K S	Assistant Professor
9	External experts	Dr. K.P.Naveenachandran	Dept, of Mathematics CUSAT
10	External experts	Dr.Vinod Kumar P.B.	Head of the Dept. of Mathematics Rajagiri School of Engineering
11	Expert from Industry	P. Padmanabhan	Rtd.Scientist(ISRO)
12	Alumni Member	Dr. N.V.Assia	Head and professor of Mathematics INA,Ezhimala,Kannur
13	University Nominee		

REGULATIONS
POST GRADUATE PROGRAMMES
MAHARAJA'S COLLEGE
(Government Autonomous)

UNDER CREDIT SEMESTER SYSTEM, 2022
(MC-PGP-CSS 2022)

REGULATIONS OF THE POST GRADUATE PROGRAMMES UNDER CREDIT SEMESTER SYSTEM, 2022 (MC-PGP-CSS 2022)

1. SHORT TITLE

1.1. These Regulations shall be called Maharaja's College (Government Autonomous) Regulations(2022) governing Post Graduate Programmes under Credit Semester System (MC-PGP-CSS 2022)

1.2. These Regulations shall come into force from the Academic Year 2022- 2023.

2. SCOPE

2.1. The regulation provided herein shall apply to all Post- graduate programmes from the academic year 2022-2023 admission.

2.2. The provisions herein supersede all the existing regulations for the regular post-graduate programmes conducted in Maharaja's College unless otherwise specified.

3. DEFINITIONS

3.1. 'Academic Committee' means the Committee constituted by the Principal under this regulation to monitor the running of the Post- Graduate programmes under the Credit Semester System (MC-PGP- CSS 2022).

3.2 'Academic Week' is a unit of five working days in which distribution of work is organized from day one to day five, with five contact hours of one hour duration on each day. A sequence of minimum of 18 such academic weeks constitute a semester.

3.2. 'Audit Course' is a course for which no credits are awarded.

3.3. 'CE' means Continuous Evaluation (Internal Evaluation)

3.4. 'College Co-ordinator' means a teacher from the college nominated by the College Council to look into the matters relating to MC-PGP-CSS 2022 for programmes conducted in the College.

3.5. 'Comprehensive viva-voce' means the oral examinations conducted by the appointed examiners and shall cover all courses of study undergone by a student for the programme.

3.6. 'Common Course' is a core course which is included in more than one programme with the same course code.

3.7. 'Core course' means a course which cannot be substituted by any other course.

3.8. 'Course' means a segment of subject matter to be covered in a semester. Each Course is to be designed variously under lectures / tutorials / laboratory or fieldwork /seminar / project / practical training / assignments / viva-voce etc., to meet effective teaching and learning needs.

3.9. 'Course Code' means a unique alpha numeric code assigned to each course ofa programme.

3.10. 'Course Credit' One credit of the course is defined as a minimum of one hour lecture /minimum of 2 hours lab/field work per week for 18 weeks in a Semester. The course will be considered as completed only by conducting the final examination.'

3.11. 'Course Teacher' means the teacher of the institution in charge of the course offered in the programme.

3.12. 'Credit (Cr)' of a course is a numerical value which depicts the measure of the weekly unit of work assigned for that course in a semester.

3.13. 'Credit point (CP)' of a course is the value obtained by multiplying the grade point (GP) by the Credit (Cr) of the course CP = GP x Cr.

3.14. 'Cumulative Grade point average' (CGPA) is the value obtained by dividing the sum of credit points of all the courses taken by the student for the entire programme by the total number of credits and shall be rounded off to two decimal places. CGPA determines the overall performance ofa student at the end ofa programme.

(CGPA = Total CP obtained / Total credits of the programme)

3.15. 'Department' means any teaching Department in the college.

3.16. 'Department Council' means the body of all teachers of a Department in a College.

3.17. 'Dissertation' means a long document on a particular subject in connection with the project /research/ field work etc.

3.18. 'Duration of Programme' means the period of time required for the conduct of the programme. The duration of post-graduate programme shall be 4 semesters spread over two academic years.

3.19. 'Elective course' means a course, which can be substituted, by an equivalent course from the same subject.

3.20. 'Elective Group' means a group consisting of elective courses for the programme.

3.21. 'ESE' means End Semester Evaluation (External Evaluation).

3.22. 'Evaluation' is the process by which the knowledge acquired by the student is quantified as per the criteria detailed in these regulations.

3.23. 'External Examiner ' is the teacher appointed from other colleges for the valuation of courses of study undergone by the students in a College. The external examiner shall be appointed by the University.

3.24. 'Faculty Advisor' is a teacher nominated by the Department Council to coordinate the continuous evaluation and other academic activities undertaken in the Department of the College.

3.25. 'Grace Grade Points' means grade points awarded to course(s), in recognition of the students' meritorious achievements in NSS/ Sports/ Arts and cultural activities etc.

3.26. 'Grade point' (GP)-Each letter grade is assigned a 'Grade point' (GP) which is an integer indicating the numerical equivalent of the broad level of performance of a student in a course.

3.27. 'Grade Point Average (GPA)' is an index of the performance of a student in a course. It is obtained by dividing the sum of the weighted grade points obtained in the course by the sum of the weights of the Course (GPA $=\Sigma$WGP / ΣMW).

3.28. 'Improvement course' is a course registered by a student for improving his performance in that particular course.

3.29. 'Internal Examiner' is a teacher nominated by the department concerned to conduct Internalevaluation.

3.30. 'Letter Grade' or 'Grade' for a course is a letter symbol (A+,A,B+,B,C+,C,D) which indicates the broad level of performance of a student for a course.

3.31. MC-PGP-CSS 2022 means Maharaja's College (Government Autonomous) Regulations Governing Post Graduate programmes under Credit Semester System, 2022.

3.32. 'Parent Department' means the Department which offers a particular postgraduate programme.

3.33. 'Plagiarism' is the unreferenced use of other authors' material in dissertations and assignments and is a serious academic offence.

3.34. 'Programme' means the entire course of study and examinations.

3.35. 'Project' is a core course in a proramme. It means a regular project work with stated credits on which the student undergo a project under the supervision of a teacher in the parent department / any appropriate research center in order to submit a dissertation on the project work as specified. It allows students to work more autonomously to construct their own learning and culminates in realistic, student-generated products orfindings.

3.36. 'Repeat course' is a course that is repeated by a student for having failed in that course in an earlier registration.

3.37. 'Semester' means a term consisting of a minimum of 90 working days, inclusive of examinations, distributed over a minimum of 18 weeks of 5 working days each.

3.38. 'Seminar' means a lecture given by the student on a selected topic and is expected to train the student in self-study, collection of relevant matter from various resources, editing, document writing and presentation.

3.39. 'Semester Grade Point Average' (SGPA) is the value obtained by dividing the sum of credit points (CP) obtained by a student in the various courses taken in a semester by the total number of credits for the course in that semester. The SGPA shall be rounded off to two decimal places. SGPA determines the overall performance ofa student at the end of a semester (SGPA = Total CP obtained in the semester / Total Credits for the semester).

3.40. 'Tutorial' Tutorial means a class to provide an opportunity to interact with students at their individual level to identify the strength and weakness of individual students.

3.41. 'University' means Mahatma Gandhi University, Kottayam, Kerala.

3.42. 'Weight' is a numeric measure assigned to the assessment units of various components of a course of study.

3.43. 'Weighted Grade Point' (WGP) is the grade point multiplied by weight.

(WGP = GP xW).

3.44. 'Weighted Grade Point Average (WGPA)' is an index of the performance of a student in a course. It is obtained by dividing the sum of the weighted grade points by the sum of the weights. WGPA shall be obtained for CE(Continuous Evaluation) and ESE(End Semester Evaluation) separately and then the combined WGPA shall be obtained for each course.

3.45. 'Internship' means gain a professional work experience

4. ACADEMIC COMMITTEE

4.1. There shall be an Academic Committee constituted by the Principal to manage and monitor the working of MC-PGP-CSS2022.

4.2. The Committee consists of

(a) Principal

(b) Vice-Principal

(c) Secretary, Academic Council

(d) The Controller of Examinations

(e) Two Teachers nominated from among the College Council

4.3. There shall be a subcommittee nominated by the Principal to look after the day-to-day affairs of the Regulations for Post Graduate Programmes under MC-PGP-CSS2022.

5. PROGRAMME STRUCTURE

5.1. Students shall be admitted to post graduate programme under the various faculties. The programme shall include three types of courses, Core Courses, Elective Courses and Common core courses. There shall be a project with dissertation and comprehensive viva-voce as core courses for all programmes. The programme shall also include assignments / seminars / practicals etc.

5.2. No regular student shall register for more than 25 credits and less than 16 credits per semester unless otherwise specified. The total minimum credits, required for completing a PG programme is 80.

5.3. Elective courses and Groups

5.3.1. There shall be at least two and not more than four elective groups(Group A, Group B, Group C, etc.) comprising of three courses each for a programme and these elective courses shall be included either in fourth semester or be distributed among third and fourth semesters. This clause is not applicable for programmes defined by the Expert Committees of Music and Performing Arts.

5.3.2. The number of elective courses assigned for study in a particular semester shall be the same across all elective groups for the programme concerned.

5.3.3. The colleges shall select any one of the elective groups for each programme as per the interest of the students, availability of faculty and academic infrastructure in the institution.

5.3.4. The selection of courses from different elective groups is not permitted.

5.3.5. The elective groups selected by the College shall be intimated to the Controller of Examinations within two weeks of commencement of the semester in which the elective courses are offered. The elective group selected by the college for the students who are admitted in a particular academic year shall not be changed.

5.4. Project work

5.4.1. Project work shall be completed in accordance with the guidelines given in the curriculum.

5.4.2. Project work shall be carried out under the supervision of a teacher of the department concerned.

5.4.3. A candidate may, however, in certain cases be permitted to work on the project in an Industrial/Research Organization on the recommendation of the supervising teacher.

5.4.4. There shall be an internal assessment and external assessment for the project work.

5.4.5. The Project work shall be evaluated based on the presentation of the project work done by the student, the dissertation submitted and the viva-voce on the project.

5.4.6. The external evaluation of project work shall be conducted by two external examiners from different colleges and an internal examiner from the college concerned.

5.4.7. The final Grade of the project (External) shall be calculated by taking the average of the Weighted Grade Points given by the two external examiners and the internal examiner.

5.5. Assignments: Every college going student shall submit atleast one assignment as an internal component for each course.

5.6. Seminar Lecture: Every regular student shall deliver one seminar lecture as an internal component for every course with a weightage of two. The seminar lecture is expected to train the student in self-study, collection of relevant matter from the various resources, editing, document writing, and presentation.

5.7. Test Papers(Internal): Every regular student shall undergo at least two class tests as an internal component for each course with a weightage of one each. The best two shall be taken for awarding the grade for class tests.

5.8. No courses shall have more than 5 credits unless otherwise specified.

5.9. Comprehensive Viva-Voce -Comprehensive Viva-Voce shall be conducted at the end of fourth semester of the programme and its evaluation shall be conducted by the examiners of the project evaluation.

5.9.1. Comprehensive Viva-Voce shall cover questions from all courses in the programme.

5.9.2. There shall be an internal assessment and an external assessment for the comprehensive Viva-Voce.

6. ATTENDANCE

6.1. The minimum requirement of aggregate attendance during a semester for appearing at the end-semester examination shall be 75%. Condonation of shortage of attendance to a maximum of 15 days in a semester subject to a maximum of two times during the whole period of the programme may be granted by the Principal.

6.2. If a student represents his/her institution, University, State or Nation in Sports, NCC, or Cultural or any other officially sponsored activities such as college union / university union etc., he/she shall be eligible to claim the attendance for the actual number of days participated subject to a maximum 15 days in a Semester based on the specific recommendations of the Head of the Department or teacher concerned.

6.3. Those who could not register for the examination of a particular semester due to shortage of attendance may repeat the semester along with junior batches, without considering sanctioned strength, subject to the existing University Rules and Clause 7.2.

6.4. A Regular student who has undergone a programme of study under earlier regulation / Scheme and could not complete the Programme due to shortage of attendance may repeat the semester along with the regular batch subject to the condition that he has to undergo all the examinations of the previous semesters as per the MC-PGP-CSS2022 regulations and conditions specified in 6.3.

6.5. A student who had sufficient attendance and could not register for fourth semester examination can appear for the end semester examination in the subsequent years with the attendance and progress report from the Principal.

7. REGISTRATION / DURATION

7.1. A student shall be permitted to register for the programme at the time of admission.

7.2. A student who has registered for the programme shall complete the programme within a period of four years from the date of commencement of the programme.

8. ADMISSION

8.1. The admission to all regular PG programmes shall be through PG- CAP (Centralized Allotment Process) of the Maharaja's College unless otherwise specified.

8.2. The eligibility criteria for admission to PG Programmes shall be published by the Maharaja's Colleg along with the notification for admission.

9. ADMISSION REQUIREMENTS

9.1 Candidates for admission to the first semester of the PG programme through CSS shall be required to have passed an appropriate Degree Examination recognized by Mahatma Gandhi University as specified or any other examination of any recognized University or authority accepted by the Academic council of Mahatma Gandhi University as eligible thereto.

9.2 Students admitted under this programme are governed by the Regulations in force.

10. PROMOTION:

10.1. A student who registers for a particular semester examination shall be promoted to the next semester.

10.2. A student having 75% attendance and who fails to register for examination of a particular semester will be allowed to register notionally and is promoted to the next semester, provided application for notional registration shall be submitted within 15 days from the commencement of the next semester.

10.3. The medium of Instruction shall be English except programmes under faculty of Language and Literature.

11. EXAMINATIONS

11.1. There shall be End Semester Examinations at the end of each semester.

11.2. Practical examinations shall be conducted by the College at the end of each semester or at the end of even semesters as prescribed in the syllabus of the particular programme. The number of examiners for the practical examinations shall be prescribed by the Board of Studies of the programmes subjected to the approval of the Academic Council of the College.

11.3. End-Semester Examinations: The examinations shall normally be conducted at the end of each semester for regular students.

11.4. There shall be one end-semester examination of 3 hours duration for each lecture based and practical courses.

11.5. A question paper may contain short answer type/annotation, short essay type questions/problems and long essay type questions. Different types of questions shall have different weightage.

12. EVALUATION AND GRADING

12.1. Evaluation: The evaluation scheme for each course shall contain two parts; (a) End Semester Evaluation(ESE) (External Evaluation) and (b) Continuous Evaluation(CE)(Internal Evaluation). 25% weightage shall be given to internal evaluation and the remaining 75% to external evaluation and the ratio and weightage between internal and external is 1:3. Both End Semester Evaluation(ESE) and Continuous Evaluation(CE) shall be carried out using direct grading system.

12.2. Direct Grading: The direct grading for CE (Internal) and ESE (External Evaluation) shall be based on 6 letter grades (A+, A, B, C, D and E) with numerical values of 5, 4,3,2, 1 and 0 respectively.

12.3. Grade Point Average (GPA): Internal and External components are separately graded and the combined grade point with weightage 1 for internal and 3 for external shall be applied to calculate the Grade Point Average (GPA) of each course. Letter grade shall be assigned to each course based on the categorization provided in 12.15.

12.4. Internal evaluation for Regular programme: The internal evaluation shall be based on predetermined transparent system involving periodic written tests, assignments, seminars, lab skills, records, viva-voce etc.

12.5. Components of Internal (CE) and External Evaluation(ESE): Grades shall be given to the evaluation of theory / practical / project / comprehensive viva-voce and all internal evaluations are based on the Direct Grading System. Proper guidelines shall be prepared by the BoS for evaluating the assignment, seminar, practical, project and comprehensive viva- voce within the framework of the regulation.

12.6. There shall be no separate minimum grade point for internal evaluation.

12.7. The model of the components and its weightages for Continuous Evaluation(CE) and End Semester Evaluation(ESE) are shown in below:

a) For Theory (CE) (Internal)

	Components	Weightage
i.	Assignment	1
ii.	Seminar	2
iii.	Best Two Test papers	2(1 each)
Total		5

(Grades of best two test papers shall be considered. For test papers all questions shall be set in such a way that the answers can be awarded A+, A, B, C, D and E grade)

b) For theory (ESE) External Evaluation is based on the pattern of questions specified in 12.15.5

c) For Practical (CE) Internal

Components	Weightage
Written/Lab test	2
Lab involvement and Record	1
Viva	2
Total	5

(The components and the weightage of the components of the practical (Internal) can be modified by the concerned BoS without changing the total weightage 5)

d) For Practical (ESE) External

Components	Weightage
Written / Lab test	7
Lab involvement and Record	3
Viva	5
Total	15

(The components and the weightage of the components of the practical (External) can be modified by the concerned BoS without changing the total weightage 15)

e) For Project (CE) Internal

Components	**Weightage**
Relevance of the topic and analysis	**2**
Project content and presentation	**2**
Project viva	**1**
Total	**5**

(The components and the weightage of the components of the project (Internal) can be modified by the concerned BoS without changing the total weightage 5)

A two stage Internal evaluation to be followed for the fruitful completion of the project.

f) For Project (ECE) External

Components	Weightage
Relevance of the topic andanalysis	3
Project content and presentation	7
Project viva	5
Total	15

(The components and the weightage of the components of the Project (External) can be modified by the concerned BoS without changing the total weightage 15)

g) Comprehensive viva-voce

Components	Internal (CE) Weight	External (ESE) Weight
Basic knowledge and Presentation skills	1	3
Topic of interest	1	3
Knowledge of core courses	3	9
Total	5	15

These basic components can be subdivided if necessary. Total as well as component weightage shall not be changed.

12.8. All grade point averages shall be rounded to two digits.

12.9. To ensure transparency of the evaluation process, the internal assessment grade awarded to the students in each course in a semester shall be published on the notice board at least one week before the commencement of external examination.

12.10. There shall not be any chance for improvement for internal grade.

12.11. The course teacher and the faculty advisor shall maintain the academic record of each student registered for the course which shall be forwarded to the University through the Principal and a copy should be kept in the college for verification for at least two years after the student completes the programme.

12.12. External Evaluation. The external examination in theory courses is to be conducted by the University at the end of the semester. The answers may be written in English or Malayalam except those for the Faculty of Languages. The evaluation of the answer scripts shall be done by examiners based on a well-defined scheme of valuation. The external evaluation shall be done immediately after the examination preferably through Centralized Valuation.

12.13. Photocopies of the answer scripts of the external examination shall be made available to the students on request as per the rules prevailing in the College/University.

12.14. The question paper should be strictly on the basis of model question paper set and directions prescribed by the BoS.

12.15. Pattern of Questions

12.15.1. Questions shall be set to assess knowledge acquired, standard, and application of knowledge, application of knowledge in new situations, critical evaluation of knowledge and the ability to synthesize knowledge. Due weightage shall be given to each module based on content/teaching hours allotted to each module.

12.15.2. The question setter shall ensure that questions covering all skills are set.

12.15.3. A question paper shall be a judicious mix of short answer type, short essay type /problem solving type and long essay type questions.

12.15.4. The question shall be prepared in such a way that the answers can be awarded A+, A, B, C, D, E grades.

12.15.5. Weight: Different types of questions shall be given different weights to quantify their range as follows:

Sl. No.	Type of Questions	Weight	Number of questions to beanswered
1.	Short Answer type questions	1	8 out of 10
2	Short essay/ problem solving type questions	2	6 out of 8
3.	Long Essay type questions	5	2 out of 4

12.16. Pattern of question for practical. The pattern of questions for external evaluation of practical shall be prescribed by the Board of Studies.

12.17. Direct Grading System. Direct Grading System based on a 6— point scale is used to evaluate the Internal and External examinations taken by the students for various courses of study.

Grade	Grade Points	Range
A+	5	4.50 to 5.00
A	4	4.00 to 4.49
B	3	3.00 to 3.99
C	2	2.00 to 2.99
D	1	0.01 to 1.99
E	0	0.00

12.18. Performance Grading. Students are graded based on their performance (GPA/SGPA/CGPA) at the examination on a 7-point scale as detailed below.

Range	Grade	Indicator
4.50 to 5.00	A+	Outstanding
4.00 to 4.49	A	Excellent
3.50 to 3.99	B+	Very good
3.00 to 3.49	B	Good(Average)
2.50 to 2.99	C+	Fair
2.00 to 2.49	C	Marginal(pass)
up to 1.99	D	Deficient(Fail)

12.19. No separate minimum is required for internal evaluation for a pass, but a minimum C grade is required for a pass in an external evaluation. However, a minimum C grade is required for pass in a course.

12.20. A student who fails to secure a minimum grade for a pass in a course will be permitted to write the examination along with the next batch.

12.21. Improvement of Course- The candidates who wish to improve the grade / grade point of the external examination of a course / courses he/ she has passed can do the same by appearing in the external examination of the semester concerned along with the immediate junior batch. This facility is restricted to first and second semesters of the programme.

12.22. One Time Betterment Programme - A candidate will be permitted to improve the CGPA of the programme within a continuous period of four semesters immediately following the completion of the programme allowing only once for a particular semester. The CGPA for the betterment appearance will be computed based on the SGPA secured in the original or betterment appearance of each semester whichever is higher. If a candidate opts for the betterment of CGPA of a programme, he/she has to appear for the external examination of the entire semester(s) excluding practicals / project/ comprehensive viva-voce. One time betterment programme is restricted to students who have passed in all courses of the programme at the regular (First appearance).

12.23. Semester Grade Point Average (SGPA) and Cumulative Grade Point Average (CGPA) Calculations. The SGPA is the ratio of sum of the credit points of all courses taken by a student in the semester to the total credit for that semester. After the successful completion of a semester, Semester Grade Point Average (SGPA) of a student in that semester is calculated using the formula given below.

Semester Grade Point Average -SGPA (S_j) = $\Sigma(C_i \times G_i) / \Sigma(C_i$

(SGPA= Total credit Points awarded in a semester / Total credits of the semester)

Where 'Sj' is the j semester, 'Gi ' is the grade point scored by the Student in the ‘i' course 'q' is the credit of the i^{th} course.

12.24 Cumulative Grade Point Average (CGPA) of a Programme is calculated using the formula:-

Cumulative Grade Point Average (CGPA) = $\Sigma((C_i \times S_i) / \Sigma(C_i$

(CGPA= Total credit points awarded in all semesters / Total credits of the programme)

Where 'CI' is the credits for the ‘i' semester 'Si' is the SGPA for the i^{th} semester. The SGPA and CGPA shall be rounded off to 2 decimal points. For the successful completion of semester, a student shall pass all courses and score a minimum SGPA of 2.0.However, a student is permitted to move to the next semester irrespective of her/his SGPA.

13. GRADE CARD

13.1 The University under its seal shall issue to the students, a consolidated grade card on completion of the programme, which shall contain the following information.

- Name of College
- Title of the PG Programme.
- Name of the Semesters
- Name and Register Number of the student
- Code, Title, Credits and Max GPA (Internal, External & Total) of each course (theory& Practical), project, viva etc. in each semester.
- Internal, external and total grade, Grade Point (G), Letter Grade and Credit Point (P) in each course opted in the semester.
- The total credits and total credit points in each semester.
- Semester Grade Point Average (SGPA) and corresponding Grade in each semester
- Cumulative Grade Point Average (CGPA), Grade for the entire programme.

- Separate Grade card will be issued at the request of candidates and based on University Guidelines issued from time to time.
- Details of description ofevaluation process- Grade and Grade Point as well as indicators, calculation methodology ofSGPA and CGPA as well as conversion scale shall be shown on the reverse side of the grade card.

14. AWARD OF DEGREE

The successful completion of all the courses with 'C' grade within the stipulated period shall be the minimum requirement for the award of the degree.

15. MONITORING COMMITTEE

There shall be a Monitoring Committee constituted by the Vice- chancellor to monitor the internal evaluations conducted by institutions.

16. RANK CERTIFICATE

The College shall publish the list of top 10 candidates for each programme after the publication of the programme results. Rank certificate shall be issued to candidates who secure positions from 1st to 3rd in the list. Position certificate shall be issued to candidates on theirrequest.

Candidates shall be ranked in the order of merit based on the CGPA secured by them. Grace grade points awarded to the students shall not be counted for fixing the rank/position. Rank certificate and position certificate shall be signed by the Controller of Examinations.

17. GRIEVANCE REDRESSAL COMMITTEE

17.1 Department level: The College shall form a Grievance RedressalCommittee in each Department comprising of the course teacher and one senior teacher as members and the Head of the Department as Chairperson. The Committee shall address all grievances relating to theinternal assessment grades of the students.

17.2. College level: There shall be a college level Grievance Redressal Committee comprising of faculty advisor, college co-ordinator, one senior teacher and one staff council member and the Principal as Chairperson.

18. REPEAL

The Regulations now in force in so far as they are applicable to programmes offered by the College and to the extent they are inconsistent with these regulations are hereby repealed. In the case of any inconsistency between the existing regulations and these regulations relating to the Credit Semester System in their application to any course offered in a College, the latter shall prevail.

19. Credits allotted for Programmes and Courses

19.1 Total credit for each programme shall be 80.

19.2 Semester-wise total credit can vary from 16 to 25

19.3 The minimum credit of a course is 2 and maximum credit is 5.

20. **Common Course**: If a course is included as a common course in more than one programme, its credit shall be same for all programmes.

21. **Course codes**: The course codes assigned for all courses (core courses, elective courses, common courses etc.) shall be unique.

22. **Models** of distribution of courses, course codes, type of the course, credits, teaching hours for a programme are given in the following tables.
Example: Programmes with practical -Total Credits 80- Scheme of the Syllabus

Semester	Course-code	Course name	Type ofthe course	Teaching Hours Per Week	Credit	Total Credits
I	Course.code1	Name1	core	4	4	19
	Course.code2	Name2	core	4	4	
	Course.code3	Name3	core	4	4	
	Course.code4	Name4	core	3	3	
	Practical Course.code5	Name5	core	10	4	
II	Course.code6	Name6	core	4	4	20
	Course.code7	Name7	core	4	4	
	Course.code8	Name8	core	4	4	
	Course.code9	Name9	core	3	4	
	Practical- Course.code10	Name10	core	10	4	
III	Course.code11	Name11	core	4	4	20
	Course.code12	Name12	core	4	4	
	Course.code13	Name13	core	4	4	
	Course.code14	Name14	core	3	4	
	Practical Course.code15	Name15	core	10	4	
IV	Course.code16	Name16	Elective	5	3	21
	Course.code17	Name17	Elective	5	3	
	Course.code18	Name18	Elective	5	3	
	Practical- Course.code19	Name19	core	10	5	
	Project- Course.code20	Name20	core		5	
	Comprehensive viva- voce -Course.code 21	Name 21	core		2	
	Total					80

Example: Programmes without practical -Total Credits 80- Scheme of the Syllabus

Semester	Course. code	Course. name	Type of the course	Teaching Hours per week	Credit	Total Credits
I	Course.code1	Name1	core	5	4	20
	Course.code2	Name2	core	5	4	
	Course.code3	Name3	core	5	4	
	Course.code4	Name4	core	5	4	
	Course.code5	Name5	core	5	4	
II	Course.code6	Name6	core	5	4	20
	Course.code7	Name7	core	5	4	
	Course.code8	Name8	core	5	4	
	Course.code9	Name9	core	5	4	
	Course.code10	Name10	core	5	4	
III	Course.code11	Name11	core	5	4	20
	Course.code12	Name12	core	5	4	
	Course.code13	Name13	core	5	4	
	Course.code14	Name14	core	5	4	
	Course.code15	Name15	core	5	4	
IV	Course.code16	Name16	Elective	5	3	20
	Course.code17	Name17	Elective	5	3	
	Course.code18	Name18	Elective	5	3	
	Course.code19	Name19	core	5	4	
	Project- Course.code20	Name20	core	5	5	
	Comprehensive viva-voce- Course.code21	Name 21	core		2	
	Total					80

Appendix

1. Evaluation first stage – Both internal and external (to be done by the teacher)

Grade	Grade Points	Range
A+	5	4.50 to 5.00
A	4	4.00 to 4.49
B	3	3.00 to 3.99
C	2	2.00 to 2.99
D	1	0.01 to 1.99
E	0	0.00

The final Grade range for courses, SGPA and CGPA

Range	Grade	Indicator
4.50 to 5.00	A+	Outstanding
4.00 to 4.49	A	Excellent
3.50 to 3.99	B+	Very good
3.00 to 3.49	B	Good
2.50 to 2.99	C+	Fair
2.00 to 2.49	C	Marginal
Upto 1.99	D	Deficient(Fail)

Theory External (ESE)

Maximum weight for external evaluation is 30. Therefore maximum Weighted Grade Point (WGP) is 150.

Type of Question	Qn. No's	Grade Awarded	Grade point	Weights	Weighted Grade Point
Short Answer	1	A+	5	1	5
	2	-	-	-	-
	3	A	4	1	4

	4	C	2	1	2
	5	A	4	1	4
	6	A	4	1	4
	7	B	3	1	3
	8	A	4	1	4
	9	B	3	1	3
	10	-	-	-	
Short Essay	11	B	3	2	6
	12	A+	5	2	10
	13	A	4	2	8
	14	A+	5	2	10
	15	-	-	-	-
	16	-	-	-	-
	17	A	4	2	8
	18	B	3	2	6
Long Essay	20	A+	5	5	25
	21	-	-	-	-
	22	-	-	-	-
	23	B	3	5	15
			TOTAL	30	117

Calculation :

Overall Grade of the theory paper = Sum of Weighted Grade Points / Total weight 117/30 = 3.90 = Grade B

Theory – Internal (CE)

Maximum Weight for internal evaluation is 5. *ie*., maximum WGP is 25

Components	Weight (W)	Grade Awarded	Grade Point (GP)	WGP=W *GP	Overall Grade of the course
Assignment	1	A	4	4	WGP/Total weight = 24/5 =4.8
Seminar	2	A+	5	10	
Test paper 1	1	A+	5	5	
Test paper 2	1	A+	5	5	
Total	5			24	A+

Practical-External-ESE

Maximum weight for external evaluation is 15. Therefore Maximum Weighted Grade Point (WGP) is 75.

Components	Weight (W)	Grade Awarded	Grade Point(GP)	WGP=W *GP	Overall Grade of the course
Written/Lab test	7	A	4	28	WGP/Total weight = 58 / 15 = 3.86
Lab involvement & record	3	A+	5	15	
viva	5	B	3	15	
Total	15			58	B

Practical-Internal-CE

Maximum weight for internal evaluation is 5. Therefore Maximum Weighted Grade point (WGP) is 25.

Components	Weight (W)	Grade Awarded	Grade Point(GP)	WGP=W *GP	Overall Grade of the course
Written/ Labtest	2	A	4	8	WGP/Total weight =17/5=3.40
Lab involvement & record	1	A+	5	5	
viva	2	C	2	4	
Total	5			17	B

Project-External-ESE

Maximum weight for external evaluation is 15. Therefore Maximum Weighted Grade Point (WGP) is 75.

Components	Weight (W)	Grade Awarded	Grade Point(GP)	WGP=W *GP	Overall Grade of the course
Relevance of the topic & Analysis	2	C	2	4	WGP/Total weight = 59/15=3.93
Project content & presentation	8	A+	5	40	
Project viva- voce	5	B	3	15	
Total	15			59	B

Project-Internal-CE

Maximum weight for internal evaluation is 5. Therefore Maximum Weighted Grade Point (WGP) is 25.

Components	Weight (W)	Grade Awarded	Grade Point(GP)	WGP=W *GP	Overall Grade of the course
Relevance of the topic & Analysis	2	B	3	6	WGP/Total weight = 21/5 = 4.2
Project content & presentation	2	A+	5	10	
Project viva-voce	1	A+	5	5	
Total	5			21	A

Comprehensive viva-voce-External-ESE.
Maximum weight for external evaluation is 15. Therefore Maximum Weighted Grade Point (WGP) is 75.

Comprehensive viva voce-Internal-CE
Maximum weight for internal evaluation is 5. Therefore Maximum Weighted Grade Point (WGP) is 25.

Components	Internal (CE) Weight	External (ESE) Weight
Basic knowledge and Presentation skills	1	3
Topic of interest	1	3
Knowledge of core courses	3	9
Total	5	15

These basic components can be subdivided if necessary

2. Evaluation - second stage -
Consolidation of the Grade(GPA) of a Course PC-I.
The End Semester Evaluation(ESE) (External evaluation) grade awarded for the course PC-I is A and its Continuous Evaluation(CE)(Internal Evaluation)grade is A. The consolidated grade for the course PC-I is as follows:

Evaluation	Weight	Grade awarded	Grade Points awarded	Weighted Grade Point
External	3	A	4.20	12.6
Internal	1	A	4.40	4.40
Total	4			17
Grade of a course.	GPA of the course =Total weighted Grade Points/Total weight 17/4 =4.25 = Grade A			

3. Evaluation -Third Stage
Semester Grade Point Average (SGPA).

Course code	Title of the course	Credits (C)	Grade Awarded	Grade Points(G)	Credit Points (CP=C X G)
01	PC-1	5	A	4.25	21.25
02	-----	5	A	4.00	20.00
03	-----	5	B+	3.80	19.00
04	-----	2	A	4.40	8.80
05	-----	3	A	4.00	12.00
TOTAL		20			81.05
SGPA	Total credit points/Total credits=81.05/20=4.05= Grade- A				

4. Evaluation - fourth Stage -

Cumulative Grade Point Average (CGPA)

If a candidate is awarded three A+ grades in semester 1 (SGPA of semester 1), semester 2 (SGPA of semester 2) and semester 4 (SGPA of semester 4) and a B grade in semester 3 (SGPA of semester 3). Then the CGPA is calculated as follows:

Semester	Credit of the Semesters	Grade Awarded	Grade point (SGPA)	Credit points
I	20	A+	4.50	90
II	20	A+	4.60	92
III	20	B	3.00	60
IV	20	A+	4.50	90
TOTAL	80			332
CGPA= Total credit points awarded / Total credit of all semesters = 332 / 80 = 4.15 (Which is in between 4.00 and 4.49 in 7-point scale)Therefore the overall Grade awarded in the programme is A				

M. Sc. MATHEMATICS PROGRAMME
MRMATPG

UNDER
CREDIT SEMESTER SYSTEM
(MC-PGP-CSS)
(Revised Autonomy Syllabus 2022 onwards)

Maharaja’s College, Ernakulam
M.Sc. Mathematics - PROGRAMME STRUCTURE

Semester	Course. code	Course. name	Type of the course	Teaching hours per week	Credit	Total Credits
I	PG1MATC01	LINEAR ALGEBRA	core	5	4	20
	PG1MATC02	TOPOLOGY I	core	5	4	
	PG1MATC03	REAL ANALYSIS	core	5	4	
	PG1MATC04	ABSTRACT ALGEBRA	core	5	4	
	PG1MATC05	GRAPH THEORY	core	5	4	
II	PG2MATC06	FUNCTIONAL ANALYSIS	core	5	4	20
	PG2MATC07	TOPOLOGY II	core	5	4	
	PG2MATC08	MEASURE THEORY AND INTEGRATION	core	5	4	
	PG2MATC09	ADVANCED ABSTRACT ALGEBRA	core	5	4	
	PG2MATC10	NUMERICAL ANALYSIS WITH PYTHON	core	5	4	
III	PG3MATC11	SPECTRAL THEORY	core	5	4	20
	PG3MATC12	COMPLEX ANALYSIS	core	5	4	
	PG3MATC13	ANALYTIC NUMBER THEORY	core	5	4	
	PG3MATC14	OPTIMIZATION TECHNIQUES	core	5	4	
	PG3MATC15	PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS	core	5	4	
IV		Elective 1	Elective	5	3	20
		Elective 2	Elective	5	3	
		Elective 3	Elective	5	3	
	PG4MATC16	MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS	core	5	4	
	Project PG4MATD01		core	5	5	
	Comprehensive viva-voce PG4MATV01		core		2	
	Total					80

ELECTIVES

Group	Course. code	Course. name	Type of the course	Teaching hours per week	Credit	Total Credits
A	PG4MATE01	ALGORITHMIC GRAPH THEORY	Elective 1	5	3	9
	PG4MATE02	COMBINATORICS	Elective 2	5	3	
	PG4MATE03	QUEUEING THEORY	Elective 3	5	3	
B	PG4MATE04	ADVANCED COMPLEX ANALYSIS	Elective 1	5	3	9
	PG4MATE05	STOCHASTIC PROCESSES	Elective 2	5	3	
	PG4MATE06	FRACTAL GEOMETRY	Elective 3	5	3	
C	PG4MATE07	DIFFERENTIAL GEOMETRY	Elective 1	5	3	9
	PG4MATE08	THEORY OF WAVELETS	Elective 2	5	3	
	PG4MATE09	ORDINARY DIFFERENTIAL EQUATIONS	Elective 3	5	3	

Maharaja's College, Ernakulam

Post Graduate Programme Outcome (PO)

At the completion of the Post Graduate Programme, the student will be able to accomplish the following

1. **Critical and creative thinking:**
    - Enables to evaluate information and its sources critically. .
    - Engage the imagination to explore new possibilities.
    - Formulate and articulate ideas.
    - Identify, evaluate and synthesize information (obtained through library, world-wide web, and other sources as appropriate) in a collaborative environment.

2. **Synergetic work culture and effective communication:**
    - Enables to develop a synergistic working relationship, which is essential for achieving a higher quantity and quality output.
    - Help to increase team productivity, enhanced individual performance and better customer engagement.

3. **Social Consciousness:**
    - Enables to understand one's role , status, rights and responsibilities as a social being which is essential for the society
    - Helps to employ the knowledge and methodologies acquired to better understand economic, legal, and social issues and act effectively.

4. **Subject knowledge:**
    - Possess breadth and depth of knowledge within their discipline and more particularly within their chosen specialization.
    - They can articulate their interpretations with an awareness and curiosity for other people's perspectives.

5. **Lifelong learning:**
    - Recognize the need for, and have the preparation and ability to engage in independent and lifelong learning in the broadest context of technological change.
    - Understands his or her learning preferences and knows how to adapt them to maximize learning under different circumstances.

6. **Multidisciplinary approach:**
    - Brings pragmatism and flexibility, allowing students to carve their path.
    - Develop knowledge in a specific topic to instill in students the ability to assess information and apply it to real-life situations.

MAHARAJA'S COLLEGE, ERNAKULAM
M.Sc. MATHEMATICS DEGREE PROGRAMME
Programme Specific Outcome (PSO)

Upon completion of M.Sc. Mathematics the graduates will be able to:

PSO No	Programme Specific Outcome	PSO Mapped to PO
1	Provide high quality education in higher mathematics committed to excellence in research	1,3,4,6
2	Find the ideal solution for a problem accurately	3,4,5
3	Investigate a situation by collecting and analyzing the data and react to it timely	3,5
4	Innovate, invent and solve complex mathematical problems using the knowledge of pure and applied mathematics	1,4
5	Transform the physical problems into mathematical models	1,2,4,6
6	Explain the knowledge of contemporary issues in the field of Mathematics and applied science	3,4,6
7	Adjust themselves to the demands, by absorbing information from society, and make connections to the growing field of mathematics & the wider world.	2,5

SEMESTER I

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FIRST SEMESTER
PG1MATCO1 - LINEAR ALGEBRA

(5 hours/week) **(Credit:4)**
(Total Hours:90) **(**Maximum Weight: 30**)**

Course Outcome:
Upon the successful completion of the course students will be able to:

CO No.	Course Outcome	Cognitive Level	CO mapped to PSO No
1	Identify the vector spaces and find its basis	U	1,4
2	Associate matrices with Linear transformations	E	3,4,6
3	Compute determinants of matrices of any order	Ap	2,4
4	Evaluate eigen values and eigenvectors of linear transformations	R,E	2,4
5	Check whether a linear transformation is Triangulable and Diagonalizable	An	2,4
6	Solve various real life problems using linear algebra	Ap,C	4,5,7

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : Kenneth Hoffman / Ray Kunze (Second Edition), *Linear Algebra*, Prentice Hall of India Pvt. Ltd., New Delhi, 1992.

Module	Content	Content Mapped to CO No	Hours
I	**Chapter 2 : Vector Spaces** 2.1 Vector Spaces 2.2 Subspaces 2.3 Basis and dimension 2.4 Coordinates 2.5 Summary of row equivalence (Proof of theorems in 2.1,2.2,2.3 are excluded)	 1 1 1 1,2 1,2	(15 hours.)
II	**Chapter 3: Linear Transformations** 3.1 Linear transformations 3.2 The algebra of linear transformations 3.3 Isomorphism 3.4 Representation of transformations by matrices 3.5 Linear functionals 3.6 The double dual 3.7 The transpose of a linear transformation	 2,6 2 2 2,6 2,6 2 2	(30 hours.)

III	**Chapter 5: Determinant** 5.1 Commutative Rings 5.2 Determinant functions 5.3 Permutation and the uniqueness of determinants 5.4 Additional properties of determinants.	 3 3,6 3,6 3,6	(18 hours.)
IV	**Chapter 6: Elementary canonical forms** 6.1 Introduction 6.2 Characteristic values 6.3 Annihilating polynomials 6.4 Invariant subspaces 6.6 Direct sum decompositions	 4 2,4,5,6 2,4,5 2,5 4	(27 hours.)

References

[1] **P. R. Halmos**: Finite Dimensional Vector spaces; Narosa Pub House, New Delhi; 1980.

[2] **A. K. Hazra**: Matrix: Algebra, Calculus and generalised inverse- Part I; CambridgeInternational Science Publishing; 2007.

[3] **I. N. Herstein**: Topics in Algebra; Wiley Eastern Ltd Reprint; 1991.

[4] **S. Kumaresan**: Linear Algebra-A Geometric Approach; Prentice Hall of India; 2000.

[5] **S. Lang**: Linear Algebra; Addison Wesley Pub.Co.Reading, Mass; 1972.

[6] **R. R. Stoll and E.T.Wong**: Linear Algebra; Academic Press International Edn; 1968.

[7] **G. Strang**: linear algebra and its applications(4th Edn.); Cengage Learning; 2006.

[8] **Klaus Jonich.** Linear Algebra, Springer Verlag.

[9] **Paul R. Halmos,** Linear Algebra Problem Book, The Mathematical Association of America.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**0**
Module II	**3**	**1**	**2**
Module III	**1**	2	**1**
Module IV	**3**	**3**	**1**
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
First Semester
Programme : M.Sc. Mathematics
PG1MATC01 : Linear Algebra

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

(Answer any ***eight*** *questions. Each question carries Weight* ***1****)*

1. Are the vectors $\alpha_1 = (1,1,2,4), \alpha_2 = (2,-1,-5,2), \alpha_3 = (1,-1,-4,0), \alpha_4 = (2,1,1,6)$ linearly independent in R^4. Find a basis for the subspace of R^4 spanned by $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

2. Let P be an invertible $n \times n$ with entries in the field F. Prove that columns of P form a basis for $F^{n\times 1}$.

3. Let W_1 and W_2 be two subspaces of R^3 defined by $W_1 = \{(x,y,z)|x+y+z=0\}, W_2 = \{(x,y,z)|x=0\}$. Find the dimension of $W_1 + W_2$.

4. True or False : There is a linear transformation $T : R^2 \to R^2$ such that $T(2,2) = (8,-6); T(5,5) = (3,-2)$. Justify.

5. Let T, U be two linear operators on a vector space V. Prove that UT is a linear operator on V.

6. Let F be a field and let f be the linear functional on F^2 defined by $f(x_1, x_2) = ax_1 + bx_2$. Find $T^t f$, where $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$.

7. Let $A = [A_{ij}]$ be a 2×2 matrix over K, a commutative ring with unity. Prove that $D(A) = A_{11}A_{22} - A_{12}A_{21}$

8. Let λ is an eigenvalue of T. Prove that λ^2 is an eigen value of T^2.

9. Show that every square matrix A over R need not have a characteristic value.

10. Let T be a linear operator on V and U be any linear operator on V such that $TU = UT$. Show that range and null space of U are invariant subspaces of T.

$(8 \times 1 = 8$ weight)

Turn Over

Part B

Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Suppose P is an $n \times n$ invertible matrix over F. Let V be an n-dimensional vector space over F, and let $\mathbb{B}$ be an ordered basis of V. Prove that there is a unique ordered basis $\mathbb{B}'$ of V such that $(i)[\alpha]_{\mathbb{B}} = P[\alpha]_{\mathbb{B}'}$ $(ii)[\alpha]_{\mathbb{B}'} = P[\alpha]_{\mathbb{B}}$.

12. Let R be a non-zero row-reduced echelon matrix. Prove that the non-zero row vectors of R form a basis for the row space of R.

13. Let V be an n-dimensional vector space over the field F, and let W be an m - dimensional vector space over F. Prove that the space $L(V, W)$ is finite-dimensional and has dimension mn.

14. Find all 2-linear functions on 2×2 matrices over a commutative ring with identity K. Also prove that the determinant function on 2×2 matrix is unique.

15. Compute the determinant of the matrix $\begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 2 & 0 & 2 \\ 4 & 1 & -1 & -1 \\ 1 & 2 & 3 & 0 \end{bmatrix}$.

16. Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Check whether T is diabonalizable.

17. Let W be an invariant subspace for T. Show that characteristic polynomial for the restriction operator T_W divides the characteristic polynomial for T. Also show that the minimal polynomial for T_W divides the minimal polynomial for T

18. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

($6 \times 2 = 12$ weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. (i) State and prove Rank- Nullity theorem.
    (ii)If A is an $m \times n$ matrix with entries in the field F. Prove that row rank(A) = column rank(A).

20. (i) Let V be a finite dimensional vector space over the field F. For each vector α in V define $L_\alpha(f) = f(\alpha)$, f in V^*. Prove that $\alpha \mapsto L_\alpha$ is an isomorphism from V onto V^{**}.
(ii)If f is a non-zero linear functional on the vector space V, then prove that the null space of f is a hyper space in V. Conversely prove that every hyper space in V is a null space of a non-zero linear functional on V.

21. (i)Let K be a commutative ring with identity and let n be a positive integer. Prove that there exist atleast one determinant function on $K^{n\times n}$
(ii)Let A be an $n \times n$ matrix over a commutative ring with identity K. Prove that A is invertible over K if and only if $detA$ is invertible in K.

22. (i) State and prove Cayley Hamilton theorem.
(ii)Let V be a finite- dimensional vector space over the field F and T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2)...(x - c_k)$ where $c_1, c_2, ...c_k$ are distinct elements of F.

$(2 \times 5 = 10$ weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FIRST SEMESTER

PG1MATC02 – TOPOLOGY - I

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Explain the concepts in Metric space and Topology	R,U	1,6
2	Evaluate a function is continuous or not.	E	1,2,4
3	Use continuous functions to understand structure of topological spaces	U	1,3,4
4	Compare the concepts in topology to metric space	An,E	1,3,6
5	Examine whether a topological space has the properties like compactness , Lindeloff and Connectedness	An,Ap	1,3

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text 1 : Walter Rudin, Principles of Mathematical Analysis (Third edition), International Student Edition.

Text 2 : K.D. Joshi, Introduction to General Topology (Revised), Wiley Eastern Ltd. 1984.

Module	Contents	Contents mapped to CO	Hours
I	**TEXT 1** A quick revision of the ideas given in the first chapter of the Prescribed text book. **Chapter 2. Basic Topology** Metric Spaces Compact Sets Perfect Sets Connected Sets **(Sections 2.15 to 2.47)**	 1,4 1,4 1,4 1,4	(20 hrs)
II	**TEXT 2** **Chapter 4. Topological Spaces (Text 2)** 1. Definition of a topological space 2. Examples of topological spaces 3. Bases and Sub-bases 4. Subspaces	 1,4 1,4 1,4 1,4	(25 hrs)

III	**TEXT 2** **Chapter 5. Basic concepts (Text 2)** 1. Closed sets and closure 2. Neighborhood, Interiorand Accumulation points (2.11 and 2.12 excluded.) 3. Continuity and related concepts 4. Making functions Continuous, Quotient Spaces [Condition 4 of Theorem 3.2 excluded)	1,4 1,4 1,2,3 1,2,3	(25 hrs)
IV	**TEXT 2** **Chapter 6. Spaces with special properties**: 1. Smallness condition on a space 2. Connectedness 3. Local connectedness and paths. (Sections 3.1 to 3.8)	1,4,5 1,4,5 1,5	(20 hrs)

References

[1] Dugundji, Topology, Universal Book Stall, New Delhi

[2] Anatolij T. Fomenko, Visual Geometry and Topology Springer-Verlag 1994

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[7] George F . Simmons, Topology and Modern Analysis, McGraw-Hill Book Company, International edition, 1963

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[9] Michael Starbird, Francis Su., Topology Through Inquiry: 58 American Mathematical Society MAA Press., 2019.

[10] Stephan Willard General Topology, Addison-Wesley, 1970.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short questions	Short essays	Long essays
Module I	2	2	1
Module II	3	2	1
ModuleIII	3	2	1
ModuleIV	2	2	1
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
First Semester
Programme : M.Sc. Mathematics
PG1MATC02 :TOPOLOGY I

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Prove that a set E in a metric space is open if and only if its complement is closed.
2. True or False: Arbitrary intersection of open sets is open. Justify.
3. Distinguish between base and subbase for a topology.
4. Define co-countable topology on a set.
5. Give examples of two topologies on a set such that one topology is weaker than the other.
6. Define closure of a subset of a topological space. Prove that if a set is closed, then its closure will be itself.
7. Define divisible property and give an example for divisible property.
8. Define Weak topolgy determined by family of functions $\{f_i : X \to Y_i : i \in I\}$.
9. State the Lebesgue covering lemma.
10. Is the union of any two connected sets connected? Justify

($8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Suppose $K \subset Y \subset X$. Prove that K is compact relative to X if and only if K is compact relative to Y.

Turn Over

12. Explain Cantor set. Show that Cantor set is perfect.

13. Prove that second countability is a hereditary property.

14. Let X be a set, τ a topology on X and S a family of subsets of X. Prove that S is a sub-base for τ iff S generates τ.

15. Prove that every open, surjective map is a quotient map.

16. Prove that every continuous image of a compact space is compact.

17. Prove that every path connected space is connected.

18. Let $f : X \to Y$ be a continuous surjection. Prove that if X is connected then Y is also connected.

($6 \times 2 = 12$ weight)

Part C

Long Essay Questions/Problems

(Answer any ***two*** *questions. Each question carries Weight* ***5****)*

19. (i) Prove that every $k-$ cell is compact.
(ii) State and prove Weierstrass theorem.

20. (i) If a space is second countable then prove that every open cover of it has a countable subcover
(ii) Let X be a space and $A \subseteq X$. Prove that interior of A is the union of all open sets contained in A.

21. (i) Prove that every second countable space is first countable. .
(ii) Every continuous real valued function on a compact space is bounded and attains its extrema.

22. (i) Prove that every closed and bounded interval is compact.
(ii) Let X_1, X_2 be two connected topological spaces and $X = X_1 \times X_2$ with the product topology then prove that X is connected.

($2 \times 5 = 10$ weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FIRST SEMESTER
PG1MATC03 - REAL ANALYSIS

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO No
1.	Identify functions of bounded variation and rectifiable paths with the ideas of total variations.	R	3,4
2.	Learn the theory of Riemann-Stieltjes integrals, to evaluate integrals of functions over increasing functions.	U,E	1,2,4
3.	Look at power series as approximations of certain functions.	An	1,6
4.	Develop the skill to reflect on problems related to convergence of functions in the field of real analysis.	C	6,7
5.	Build up an intuitive idea of continuity, differentiation, and integration through calculations and solving application problems.	Ap,C	4,5

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text 1 : **Tom Apostol, Mathematical Analysis (second edition), Narosa Publishing House.**

Text 2: **Walter Rudin, Principles of Mathematical Analysis (Third edition), International Student Edition.**

Pre-requisites : A quick review on continuity, uniform continuity, convergence of sequence and series. (No question shall be asked from this section.)

Module	Contents	Contents Mapped to CO	Hours
I	**Text 1** **Chapter 6 : Functions of bounded variation and rectifiable curves** 6.1 Introduction 6.2 Properties of monotonic functions 6.3 Functions of bounded variation 6.4 Total variation 6.5 Additive property of total variation 6.6 Total variation on [a, x] as a functions of x 6.7 Functions of bounded variation expressed as the difference of increasing functions 6.8 Continuous functions of bounded variation 6.9 Curves and paths 6.10 Rectifiable path and arc length 6.11 Additive and continuity properties of arc length 6.12 Equivalence of paths, change of parameter.	1, 5	(20 hrs.)
II	**Text 2** **Chapter 6 : The Riemann-Stielljes Integral** 6.1 - 6.11 Definition and existence of the integral 6.12 - 6.19 Properties of the integral 6.20 - 6.22 Integration and differentiation 6.23 - 6.25 Integration of vector valued functions.	2, 5	(25 hrs.)
III	**Text 2** **Chapter 7 : Sequence and Series of Functions** 7.1 – 7.6 Discussion of main problem 7.7 – 7.10 Uniform convergence 7.11 – 7.15 Uniform convergence and continuity 7.16 Uniform convergence and integration 7.17 - 7.18 Uniform convergence and differentiation 7.26 The Stone-Weierstrass theorem (without proof).	4, 5	(25 hrs)
IV	**Text 2** **Chapter 8 : Some Special Functions** 8.1 – 8.5 Power series 8.6 The exponential and logarithmic functions 8.7 The trigonometric functions 8.8 The algebraic completeness of complex field 8.9 – 8.16 Fourier series.	3, 5	(20 hrs)

References

[1] Royden H.L, Real Analysis, 2nd edition, Macmillan, New York.

[2] Bartle R.G, The Elements of Real Analysis, John Wiley and Sons.

[3] S.C. Malik, Savitha Arora, Mathematical Analysis, New Age International Ltd.

[4] Edwin Hewitt, Karl Stromberg, Real and Abstract Analysis, Springer International, 1978.

Question paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short questions	Short essays	Long essays
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	10	4

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
First Semester
Programme - M.Sc. Mathematics
PG1MATC03 - REAL ANALYSIS

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions/Problems
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Define the function of bounded variation on $[a,b]$. Show that if f is monotonic on $[a,b]$ then f is of bounded variation on $[a,b]$.
2. Check whether the function $f(x)= x^2 \cos\frac{1}{x}, x \neq 0$ and $f(0)=0$ is of bounded variation on [0,1].
3. Let f and g be complex– valued functions defined as $f(t)= e^{\pi i t}$ if $t \in [0,1]$, $g(t)= e^{\pi i t}$ if $t \in [0,2]$. Prove that the length of g is twice that of f.
4. Define Riemann –Stieltjes integral of f with respect to α over $[a,b]$. If f is continuous on $[a,b]$ then prove that $f \in \mathcal{R}(\alpha)$ on $[a,b]$.
5. Let f be Riemann integrable on $[a\ b]$ and for $a \leq x \leq b$, let $F(x)=\int_a^x f(t)dt$. Prove that F is continuous on $[a,b]$.
6. If $f \in \mathcal{R}(\alpha)$ on $[a,b]$ and if $|f(x)| \leq M$ on $[a,b]$ then prove that $\left|\int_a^b f d\alpha\right| \leq M[\alpha(b)-\alpha(a)]$.
7. Differentiate between uniform convergence and point wise convergence with examples.
8. State Stone- Weierstrass theorem. Prove that $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$
9. Let $\{f_n\}$ be a sequence of real valued differentiable functions that converge to f. Is it true that $f_n' \to f'$? Justify
10. For m,n = 1,2,3 ... let $S_{m,n}=\frac{m}{m+n}$. Show that $\lim_{m\to\infty}\lim_{n\to\infty} S_{m,n} \neq \lim_{n\to\infty}\lim_{m\to\infty} S_{m,n}$

(8x1= 8 Weight)

Part B
Short Essay Questions/Problems
*(Answer any **six** questions. Each question carries Weight **2**)*

11. Let f be a defined on $[a,b]$. Prove that f is of bounded variation on $[a,b]$ if and only if f can be expressed as the difference of two increasing functions.
12. If $\in (a,b)$ then show that $\wedge_f(a,b)= \wedge_f(a,c)+\wedge_f(c,b)$.
13. Suppose $f \in \mathcal{R}(\alpha)$ on $[a\ b]$, $m \leq f \leq M$, ϕ is continuous on $[m\ M]$ and $h(x)= \phi(f(x))$ on $[a,b]$ then prove that $h \in \mathcal{R}(\alpha)$ on $[a,b]$.

Turn Over

14. Let $\{f_n\}$ be a sequence of functions defined on E such that $|f_n(x)| \leq M_n$ for all $n = 1,2,3, \ldots$ and $x \in E$. Prove that $\sum f_n$ converges uniformly on E if $\sum_n M_n$ converges.

15. Let α be monotonically increasing function on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$ for $n = 1,2,3, \ldots,$ and suppose $f_n \to f$ uniformly on $[a, b]$ Prove that $f \in \boldsymbol{\mathcal{R}(\alpha)}$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$

16. If $f \in \boldsymbol{\mathcal{R}(\alpha)}$ on [a,b] Prove that $|f| \in \boldsymbol{\mathcal{R}(\alpha)}$ on [a,b] Prove that $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha$

17. If f is Riemann integral on [a,b] for $a \leq x \leq b$ put $F(x) = \int_a^b f(t)dt$, then prove that F is continuous on [a,b]. Also, prove that if f is continuous at $x_0 \in$ [a,b] , then F is differentiable at x_0 and $F'(x_0) = f(x_0)$

18. Prove that $\lim_{x\to+\infty} x^{-\infty} log x = 0$ (6x2=12 Weight)

Part C

Long Essay Questions/Problems

(Answer any two questions. Each question carries Weight 5)

19. Let f be of bounded variation on $[a, b]$ if $x \in [a,b]$ let $V(x) = V_f(a, x)$ and put $V(a) = 0$ then prove that every point of continuity of f is also a point of continuity of V and the converse is also true.

20. a) If $f \in \boldsymbol{\mathcal{R}(\alpha)}$ and $g \in \boldsymbol{\mathcal{R}(\alpha)}$ then prove that $fg \in \boldsymbol{\mathcal{R}(\alpha)}$.

    b) Suppose α increases monotonically and $\alpha' \in \boldsymbol{\mathcal{R}}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Prove that $f \in \boldsymbol{\mathcal{R}(\alpha)}$ if and only if $f\alpha' \in \boldsymbol{\mathcal{R}}$. Also show that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

21. a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

    b) Let $\{f_n\}$ be the sequence of functions on $(0, \infty)$ defined by $f_n(x) = \frac{nx}{1+n^2x^2}$. Show that $\{f_n\}$ is not uniformly convergent.

22. a) Prove that every trigonometric polynomial is periodic, with period 2π

    b) Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum a_n x^n = \sum b_n x^n$. Show that if E has a limit point in S then a_n=b_n for all $x \in S, n = 0,1,2, \ldots$

(2x5=10 Weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FIRST SEMESTER
PG1MATC04 – ABSTRACT ALGEBRA

(5 hours/week) **(Credit : 4)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	CourseOutcome	Cognitive Level	CO Mapped to PSO
1	Understand the concepts of Group,ring, field and related areas	U	1,6
2	Analyze and identify the structures and mappings	An	1,2,4,6
3	Perform computations	E	3,5,7
4	Apply theorems and concepts	Ap	2,3,4,5
5	Invent new development	C	1,4

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson Education.

Pre Requisite: Group,Ring Field, Integral Domain etc

Module	Contents	Contents mapped to CO	Hours
I	**Part II** Section 9 : 9.1-9.3 Orbits Section 11 : Direct products and finitely generated Abelian groups Section 12 : Plane Isometries **Part III** Section 14 : Factor Group	1,3 4,5 2,5 1,2,3	(20 hours)
II	**Part III** Section 16 : Group Action on a set Section 17 : Application of G-sets to counting **Part IV** Section 20 : Fermat's and Euler's Theorems	1,5 4 2,3,4	(20 hours)

III	**Part VII** Section 34 : Isomorphism Theorems Section 36 : Sylow's theorems Section 37 : Applications of the sylow theory	1,2,4 3,4,5 3,4	(25 hours)
IV	**Part IV** Sections 21 : The field of Quotients of an Integral Domain Section 22 : Rings of polynomials Section 23 : Factorisation of polynomials over a field	1,2 1 3,4	(25 hours)

References

[1] I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi,1975.

[2] Hungerford, Algebra,Springer

[3] M. Artin, Algebra, Prentice -Hall of India, 1991

[4] N. Jacobson, Basic Algebra Vol. I, Hindustan PublishingCorporation

Question Paper Pattern

Module	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	2	2	0
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	2
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
First Semester
Programme - M.Sc. Mathematics
PG1MATC04 – ABSTRACT ALGEBRA

Time: Three Hours Maximum Weight: 30

Part A

Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. State Fundamental Theorem of Finitely Generated Abelian Groups.
2. Find the order of the element (3, 10, 9) in Z_4 x Z_{12} x Z_{15}
3. Explain Group Action on a set.
4. Find the number of orbits in {1,2,3,4,5,6,7,8} under the cyclic subgroup <(1,3,5,6)>?
5. State and prove Euler's theorem.
6. Define p-group with an example.
7. Define class equation of G with an example.
8. Find two Sylow 2-subgroups of S_4 and show that they are conjugate.
9. Find the sum and product of the polynomials f(x)=4x-5 and g(x)= $2x^2$- 4x + 2 in Z_8[x]
10. Justify that x^2 -2 has no Zeros in Q.

(8 x 1 = 8 Weights)

Part B

Short Essay Questions

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove that the group Z_m x Z_n is cyclic and is isomorphic Z_{mn} if and only if m and n are relatively prime.
12. Write a note on plane isometries.
13. If X be a G-set then prove that G_x is a subgroup of G for each x∈X.

14.Show that for every integer n, the number n^{33}- n is divisible by 15

15.State and prove Cauchy's theorem

16.State and prove second Sylow theorem.

17. State and prove Eisenstein Criterion.

18.Show that $f(x) = x^2 + 8x – 2$ is irreducible over Q. Is f(x) irreducible over R ? Over C ?

(6 x 2 = 12 Weights)

Part C
Long Essay Questions

(Answer any two questions. Each question carries weight 5)

19 a) Prove that the set G_n of nonzero elements of Z_n that are not zero divisors forms a group under multiplication modulo n

b) Use Fermat's theorem, find the remainder of 3^{47} when it is divisible by 23.

20 a) Let G be a group containing normal subgroups H and K such that $H \cap K=\{e\}$ and H VK= G. Show that G is isomorphic to H ×K.

b) For a prime number p, Prove that every group of order p^2 is abelian.

21. State and Prove Evaluation Homomorphisms theorem for Field theory

22. State and Prove theorem on Division Algorithm for F[x]. Also prove Factor theorem.

(2x5=10 Weights)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FIRST SEMESTER
PG1MATC05 - GRAPH THEORY

(5 hours/week) **(Credit : 4)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand the fundamentals of Graph Theory	U	**1,6**
2	Analyze and identify the structure of different types of graphs	R,An	**1,2,4,6**
3	Model various relations and processes in physical, biological, social and information systems as graphs by evaluating the situation	E	**3,5,7**
4	Apply different algorithms and results of graph theory to solve real life problems	Ap	**2,3,4,5**
5	Create new graph classes using the graph operators	C	**1,4**

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : **R.Balakrishnan and K. Ranganathan,** A Text book of Graph Theory, Springer-Verlag New York, Inc., 2000.

Module	Contents	Content Mapped to CO	Hours
I	**Chapter 1 : Basic results** 1.1 Basic concepts 1.2 Sub graphs 1.3 Degrees of vertices 1.4 Paths and connectedness 1.5 Automorphism of a simple graph 1.7 Operations on graphs **Chapter 2 : Directed graphs** 2.1 Basic concepts 2.2 Tournaments. **Chapter 3 : Connectivity** 3.1 Vertex cuts and edge cuts 3.2 Connectivity and edge connectivity 3.3 Blocks.	 1,3,5 1,2 1,4 1,4 1,4 1,4,5 1,2,3 1,2,4 1,4 1,4 1,4	(20 hrs.)

II	**Chapter 4 : Trees** 4.1 Definition, characterization and simple properties 4.2 Centres and centroids 4.3 Counting the number of spanning trees 4.4 Cayley's formula **Chapter 10 : Applications** 10.1 The Connected Problem 10.2 Kruskal's Algorithm 10.3 Prim's Algorithm	1,2,3,4 1,2 2,4 2,4 4 4 4	(20 hrs.)
III	**Chapter 5 : Independent Sets and Matchings** 5.1 Vertex independent sets and vertex coverings 5.2 Edge independent sets **Chapter 6 : Eulerian and Hamiltonian Graphs** 6.1 Eulerian Graphs 6.2 Hamiltonian Graphs **Chapter 7 :Graph Colorings** 7.1 Vertex colorings 7.2 Critical graphs 7.3 Triangle free graphs.	1,4 1,4 1,2,3,4 1,2,3,4 1,3,4,4 1,2,4 1,2,4	(25 hrs)
IV	**Chapter 7 :Graph Colorings** 7.4 Edge colorings of graphs **Chapter 8 : Planarity** 8.1 Planar and non-planar graphs, 8.2 Euler formula and its consequences 8.3 K_5 and $K_{3,3}$ are non-planar graphs 8.4 Dual of a plane graph 8.5 The four color theorem and Heawood five color theorem.	1,3,4 1,2,4 1,4,4 1,4 1,2,3,4 3,4	(25 hrs)

References

[1] **J. A. Bondy, U.S.R. Murty**: Graph Theory; Springer; 2000.

[2] **S. M. Cioaba and M.R. Murty**: A First Course in Graph Theory and Combinatorics; Hindustan Book Agency; 2009.

[3] **John Clark and Derek Allan Holton**, A First Look at Graph Theory, Allied Publishers.

[4] **R. Diestel**: Graph Theory(4th Edn.); Springer-Verlag; 2010

[5] **J. L. Gross**: Graph theory and its applications (2nd edn.); Chapman & Hall/CRC; 2005. [6]

F. Harary: Graph Theory; Narosa Pub. House, New Delhi; 1992.

[7] **Narsingh Deo**: Graph Theory with Applications to Engineering & Computer Science; Dover Publications Inc; New York

[8] **W. T. Tutte**: Graph Theory; Cambridge University Press; 2001

[9] **D. B. West**: Introductionto graph theory; Prentice Hall; 2000.

[10] **R. J. Wilson** : Introduction to Graph Theory; Longman Scientific and Technical Essex (co-published with John Wiley and sons NY); 1985.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

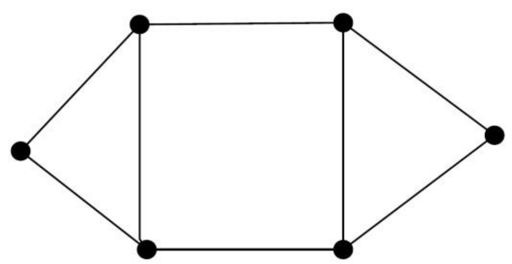
Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
First Semester
Programme - M.Sc. Mathematics
PG1MATC05 - GRAPH THEORY

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions/Problems
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. State and prove the first theorem on graph theory and also show that in any graph G, the number of vertices of odd degree is even.
2. Prove: If G is simple and $\delta > \frac{n-1}{2}$ then G is connected.
3. Prove that an edge e of a connected graph G is a cut edge of G if and only if there exists vertices u and v such that e belongs to every u-v path in G.
4. State Cayley's formula, how many spanning trees does K_4 have?
5. Show that a forest of k trees which have a total of n vertices has n−k edges.
6. Let G be a simple graph with n ≥ 3 vertices. Prove that if d(u) + d(v) ≥ n-1 for every pair of nonadjacent vertices u and v of G, then G is traceable.
7. Prove that for any graph G with n vertices and independence number α, $\frac{n}{\alpha} \leq \chi \leq$ n - α - 1
8. Prove: If G is triangle free then (G) is also triangle free.
9. Find a simple graph G with degree sequence (4, 4, 3, 3, 3, 3) such that G is planar.
10. Define dual of a graph. Find the dual of the following graph.



(8 x 1 = 8 Weights)

Part B
Short Essay Questions/Problems
*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove: The set Aut(G) of all automorphisms of a graph G is a group with respect to the composition of mappings as the group operation.
12. Explain cartesian product, strong product and tensor product of two graphs with examples. Give the relation between these three products.
13. Find all non-isomorphic trees on 7 vertices.

Turn Over

14. Explain Kruskal's Algorithm.
15. Show that in a critical graph G, no vertex cut is a clique.
16. Prove that K5 is non-planar.
17. Prove: If G is bipartite graph, $\chi'(G) = \Delta(G)$
18. State and prove Euler formula for planar graphs.

(6 x 2 = 12 Weights)

Part C
Long Essay Questions/Problems

(Answer any two questions. Each question carries Weight 5)

19. (i) Define a tournament. Prove that every tournament contains a directed Hamiltonian path.

    (ii) Connectivity and edge connectivity of a simple cubic graph are equal. Prove.

20. (i) If e is not a loop of a connected graph G, then prove τG) = τ(G - e) + τ(G.e).

    (ii) Prove that every 3-edge-connected graph G has three spanning trees whose intersection is a spanning totally disconnected sub graph of G.

    (iii) Let G be a connected graph. Prove that G^c is connected and if diam (G) ≥ 3 then diam(G^c)≤ 3.

21. Prove that a graph G is Eulerian iff each edge e of G belongs to an odd number of cycles of G.
22. Prove that every Planar graph is 5 vertex colorable.

(2x5=10 Weight)

SEMESTER II

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards

SECOND SEMESTER

PG2MATC06 – FUNCTIONAL ANALYSIS

(5 Hours/week) **(Credit 4)**

(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	CourseOutcome	Cognitive Level	CO Mapped to PSO
1	Understand the concepts of different Spaces and related areas	U	1,6
2	Analyze and identify lemmas, mappings etc	An, R	1,2,4,6
3	Perform computations	E	3,5,7
4	Apply ideas from the theory of spaces to other areas	Ap	2,3,4,5
5	Invent new development	C	1,4

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: Erwin Kreyszig, Introductory FunctionalAnalysiswith applications, John Wiley and sons, New York

Module	Contents	Contents mapped to CO	Hours
I	**Chapter 2. Normed Spaces, Banach Spaces** 2.1 Vector Space 2.2 Normed space, Banach Space 2.3 Further properties of Normed Spaces 2.4 Finite Dimensional Normed Spaces and Subspaces, 2.5 Compactness and Finite Dimension 2.6 Linear Operators 2.7 Bounded and Continuous Linear Operators	1,3 1,3 4,5 2,4 5 1,2,4 4	(20 hrs)
II	**Chapter 2. Normed Spaces, Banach Spaces** 2.8 Linear functional 2.9 Linear Operators and Functionals on finite dimensional Spaces 2.10 Normed Spaces of Operators. Dual space **Chapter 3. Inner Product Spaces. Hilbert Spaces**	1,2 3,4 1,4,5	(20hrs)

	3.1 Inner Product Space. Hilbert space 3.2 Further Properties of Inner Product Space.	1,2 3,4	
III	**Chapter 3. Inner Product Spaces. Hilbert Spaces** 3.3 Orthogonal Complements and Direct Sums 3.4 Orthonormal Sets and Sequences 3.5 Series Related to Orthonormal Sequences and Sets 3.6 Total Orthonormal Sets and Sequences 3.8 Representation of Functionals on Hilbert spaces 3.9 Hilbert Adjoint Operators 3.10 Self Adjoint, Unitary and Normal Operators	4,5 3 5 1,2,4 4,5 5 4,5	(25hrs)
IV	**Chapter 4.Fundamental Theorems for Normed and Banach Spaces** 4.1 Zorn's Lemma 4.2 Hahn- Banach Theorem, 4.3 Hahn- Banach Theorem for Complex Vector Spaces and Normed spaces 4.5 Adjoint Operator 4.6 Reflexive Spaces 4.7 Category Theorem (Statement only). Uniform Boundedness Theorem	2 4 4,5 2,5 1,5 4	(25hrs)

References

[1] Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York1963.

[2] Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi1989

[3] Somasundaram. D, Functional Analysis, S.ViswanathanPvt Ltd, Madras,1994

[4] Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut:1995-96

Question paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short questions	Short essays	Longessays
Module I	3	2	1
Module II	2	2	1
ModuleIII	3	2	1
ModuleIV	2	2	1
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**
Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Second Semester
Programme – M.Sc. Mathematics
PG2MATC06 – FUNCTIONAL ANALYSIS

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Give an example for a Normed space which is not a Hilbert space. Justify.
2. Prove that the Nullspace of a linear operator is a closed vector space.
3. Prove that the norm induced on an Inner Product space satisfies the Parallelogram equality.
4. In an Inner Product Space $x \perp y$ if and only if $\|x + \alpha y\| \geq \|x\|$ for scalar α.
5. Establish a characterization of a set in Hilbert space whose span is dense.
6. Let A and B⊃ A be non empty subsets of an inner product space X. Show that
   i) $A \subset A^{\perp\perp}$ ii) $B^{\perp} \subset A^{\perp}$ iii) $A^{\perp\perp\perp} = A^{\perp}$
7. Show that for any bounded linear operator T on a Hibert space H, the operators $T_1 = (T+T^*)/2$ and $T_2 = (T-T^*)/2i$ are self adjoint.
8. Show that the canonical mapping C defined by $C(x) = g_x$ where x ∈ , ∈ X" is an isomorphism from the Normed space X onto the normed space range of C.
9. Let S, T ∈ B(X, Y), Show that (i) $(ST)^X = T^X S^X$ and (ii) $(\alpha T)^X = \alpha T^X$
10. State Baire's Category Theorem for Complete Metric spaces.

(8x1=8 Weight)

Part B
Short Essay Questions
*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove that in a finite dimensional normed space *X*, compact sets are precisely closed and bounded.
12. Define Equivalent norms on a Vector space and prove that in a finite dimensional linear space, any two norms are equivalent.
13. Prove that in a finite dimensional normed space X, every linear operator is bounded.
14. If Y is a Banach space, then prove that B(X, Y) is a Banach Space.
15. State and prove Riesz's Theorem for bounded linear functional on Hilbert space *H*.
16. State and prove Bessel's Inequality.
17. Prove that every Hilbert space is Reflexive.
18. Let $T: X \to Y$ be a bounded linear operator, where X and Y are Normed spaces. Define the adjoint operator *T*. Show that the adjoint operator is bounded, linear and $\|T^X\| = \|T\|$. (6x2=12 Weight)

Part C
Long Essay Questions

(Answer any ***two*** *questions. Each question carries Weight* ***5****)*

19. For a normed space, prove that there is a unique Banach space $\hat{X}$ and isometry A from X into subspace W of $\hat{X}$ which is dense in $\hat{X}$.
20. Show that dual space of l^p is l^q here $1 < p < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$
21. Let *H* be a Hilbert Space Then prove that,
    i) If *H* is separable, every orthonormal set in *H* is countable.
    ii) If *H* contains an orthonormal sequence which is total in *H* , then *H* is separable.
22. State and prove Generalised Hahn Banach Theorem.

(2x5=10 Weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
SECOND SEMESTER
PG2MATC07 – TOPOLOGY II

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Identify topological spaces in which points and closed sets are separated	R	3
2	Create topological spaces using product topology	C	1
3	Check whether a topological space is embeddable in a cube	E	3,6
4	Characterize topological spaces and topological properties	An	3

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text 1 : K.D. Joshi, Introduction to General Topology (Revised), Wiley Eastern Ltd. 1984.

Module	Contents	Contents mapped to CO	Hours
I	**Chapter 7. Separation Axioms** 1. Hierarchy of separation axioms 2. Compactness and Separation Axioms. (Sections 2.1 to 2.10) 3. The Urysohn Characterisation of normality (Proof of Lemma 3.4 excluded) 4. Tietze Characterisation of normality	 1,4 1,4 1,4 1,4	(25 hrs)
II	**Chapter 8. Products and Co-products** 1. Cartesian products of families of sets (Section 1.1. to 1.7) 2. The product topology 3. Productive Properties (Sections 3.1 to 3.6)	 2,3 2,3 2,3	(20 hrs)
III	**Chapter 9. Embedding and Metrisation** 1. Evaluation functions into products 2. Embedding Lemma and Tychonoff Embedding 3. The Uryson Metrisation Theorem. **Chapter 11. Compactness** 3. Local Compactness 4. Compactifications (upto 4.7)	 3 3,4 3,4 3,4 3	(25 hrs)

IV	**Chapter 10. Nets and Filters** 1. Definition and Convergence of Nets 2. Topology and Convergence of Nets 3. Filters and Their Convergence	4 1,4 2,4	(20 hrs)

References

[1] Dugundji, Topology, Universal Book Stall, New Delhi

[2] Anatolij T. Fomenko, Visual Geometry and Topology Springer-Verlag 1994

[3] J. L Kelley, General Topology, Van Nostrand, Reinhold Co., New York,1995

[4] Steven G. Krantz, Essentials of Topology with Applications, Chapman and Hall/CRC; 1st Edn., 2017.

[5] S. Kumerasan, Topology of Metric Spaces, Alpha Science International Ltd. Harrow, UK, 2005.

[6] James R. Munkres, Topology 2 Edn, Pearson Education.

[7] George F . Simmons, Topology and Modern Analysis, McGraw-Hill Book Company, International edition, 1963

[8] I.M. Singer & J.A. Thorpe, Lecture Notes on Elementary Topology & Geometry, Springer Verlag 2004

[9] Michael Starbird, Francis Su., Topology Through Inquiry: 58 American Mathematical Society MAA Press., 2019.

[10] Stephan Willard General Topology, Addison-Wesley, 1970.

Question Paper Pattern

	Part A Weight 1 Short questions	Part B Weight 2 Short essays	Part C Weight 5 Long essays
Module I	3	2	1
Module II	2	2	1
ModuleIII	3	2	1
ModuleIV	2	2	1
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER M.Sc. DEGREE (C.S.S) EXAMINATION

Second Semester

Programme : M.Sc. Mathematics

PG2MATC07 :TOPOLOGY II

Time : Three Hours Maximum Weight : 30

Part A

Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Define regular and completely regular topological spaces.
2. Prove that in a Hausdorff space, limits of sequences are unique.
3. Prove that a compact subset in a Hausdorff space is closed.
4. Explain productive property. Give an example of a topological property which is not productive.
5. Differentiate between Cube and Hilbert Cube.
6. Prove that the evaluation function of a family of functions is one-to-one if and only if that family distinguishes points .
7. Prove that if a space X is regular and locally compact at a point $x \in X$ then x has a local base consisting of compact neighbourhoods.
8. Distinguish between Alexandroff compactification and Stone-Cech compactification of space X.
9. Define Riemann net.
10. Let $S : D \to X$ be a net and $\mathscr{F}$ the filter associated with it. Let $x \in X$. Prove that S converges to x as a net iff $\mathscr{F}$ converges to x as a filter.

($8 \times 1 = 8$ weight)

Part B

Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Show that all metric spaces are T_4.

Turn Over

12. Prove that every regular, Lindeloff space is normal.
13. Prove that if the product is non-empty,then each coordinate space is embeddable in it.
14. Define Product topology. Prove that projection functions are open
15. Describe the evaluation map of a family of functions $\{f_i : X \to Y_i : i \in I\}$.
16. State and prove embedding lemma.
17. Let $S : D \to X$ be a net in a topological space and let $x \in X$. Prove that x is a cluster point iff there exists a subnet of S which converges to x in X.
18. Prove that every filter is contained in an ultrafilter.

(6 $\times$ 2 $=$ 12 weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. State and prove Urysohn characterisation of normality.
20. (i) Prove that complete regularity is a productive property.
    (ii) Prove that connectedness is a productive property.
21. (i) Prove that a second countable space is metrisable iff it is T_3.
    (ii) Prove that one point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff.
22. (i) Let X, Y be a topological space, $x \in X$, and $f : X \to Y$ a function. Prove that f is continuous at x iff whenever a filter $\mathscr{F}$ converges to x in X, the image filter $f_{\#}(\mathscr{F})$ converges to $f(x)$ in Y.
    (ii) State and Prove Tychnoff's theorem.

(2 $\times$ 5 $=$ 10 weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
SECOND SEMESTER
PG2MATC08 - MEASURE THEORY AND INTEGRATION

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome
Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1.	Understanding the basic concepts of measure and integration theory.	U	1,6,7
2.	Acquire basic knowledge of measure theory needed to understand integration through measure theory.	R	1,7
3.	Describe and apply the notion of measurable functions and sets.	U, Ap	2,3,4
4.	Learn a Classical Banach Space which is essential for the study of functional analysis	An	1,6,7

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : H.L. Royden, Real Analysis, Third edition, Prentice Hall of India Private Limited.

Pre-requisites: Algebras of sets, the axiom of choice and infinite direct products, open and closed sets of real numbers. (Chapter 1 - section 4, 5, Chapter 2 - section 5 of Text 1).

(No questions shall be asked from this section)

Module	Contents	Contents Mapped to CO No	Hours
I	**Chapter 3 : Lebesgue measure** 1. Introduction 2. Outer measure 3. Measurable sets and Lebesgue measure 4. Non-measurable sets 5. Measurable functions.	1,3	(20 hours)
II	**Chapter 4: The Lebesgue integral** 1. The Riemann integral 2. The Lebesgue integral of a bounded function over a set of finite measure 3. The integral of a non-negative function 4. The general Lebesgue integral **Chapter 5 : Differentiation and Integration** 1. Differentiation of monotone functions.	2	(25 hours)

III	**Chapter 6 : The Classical Banach Spaces** 1. The L^p spaces 2. The Minkowski and Holder inequalities 3. Convergence and Completeness 4. Approximation in L^p 5. Bounded linear functional on the L^p spaces	3,4	(15 hours)
IV	**Chapter 11 : Measure and integration** 1. Measure spaces 2. Measurable functions 3. Integration 4. General convergence theorems 5. Signed measures 6. The Radon-Nikodym Theorem(Statement Only) **Chapter 12: Measure and Outer Measure** 1. Outer measure and measurability 2. The Extension Theorem.	2	(30 hours)

References

[1] Halmos P.R, Measure Theory, D.van Nostrand Co.

[2] P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., New Delhi, 1986(Reprint 2000).

[3] G. de Barra, Measure Theory and Integration, New Age International (P) Linnilect Publishers

[4] R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc New York, 1966.

Question paper pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short questions	Short essays	Long essays
Module I	2	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	3	2	1
Total	**10**	**8**	**4**

QP Code. **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Second Semester
Programme – M.Sc. Mathematics
PG2MATC08 - MEASURE THEORY AND INTEGRATION

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. If $m^* E = 0$, then prove that E is measurable.
2. Define measurable function. Show that if f and g are measurable, fg is also measurable.
3. Define characteristic function and Borel measurable function.
4. Let f be a nonnegative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.
5. When we say that a function is Riemann integrable.
6. Define a simple function by defining characteristic function.
7. State Minkowski's and Holder's inequality.
8. If $A, B \in \mathcal{B}$, and $A \subseteq B$, then prove that $\mu A \leq \mu B$.
9. Define a bounded linear functional and give an example
10. Define a signed measure.

(8x1=8 Weights)

Part B
Short Essay Questions

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove that Lebesque measure is invariant under translation modulo 1.
12. Prove thar every Borel set is measurable. In particular prove that each open set and each closed set is measurable
13. State and prove bounded convergence theorem.
14. State and prove Fatou's lemma and hence state and prove Monotone convergence theorem.
15. State and prove Riesz-Fischer theorem
16. Prove that a normed linear space X is complete if and only if every absolutely summable series is summable.
17. If $E_i \in \mathcal{B}, \mu E_i < \infty$ and $E_i \supset E_{i+1}$, then prove that $\mu(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} \mu E_n$.
18. State and prove Hahn decomposition theorem.

(6x2=12 Weights)

Turn Over

Part C

Long Essay Questions

(Answer any ***two*** *questions. Each question carries Weight* ***5****)*

19. Prove that the interval (a,∞) is measurable. Also prove that the outer measure of an interval is its length.

20. Let f be defined and bounded on a measurable set E with mE finite. Then prove that

$$\inf_{f \le \Psi} \int_E \psi(x)dx = \sup_{f \ge \varphi} \int_E \varphi(x)dx$$ for all simple functions φ and ψ, if and only if f is measurable

21. State and prove Riesz Representation theorem.

22. State and prove Radon-Nikodym theorem.

(2x5=10 Weights)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
SECOND SEMESTER
PG2MATC09 – ADVANCED ABSTRACT ALGEBRA

(5 hours/week) **(Credit : 4)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand the concepts Extension field and related areas	U	**1,6**
2	Analyze and identify the structures and mappings	R, An	**1,2,4,6**
3	Perform computations and constructions	R, E	**3,5,7**
4	Apply theorems and concepts	Ap	**2,3,4,5**
5	Invent new development	C	**1,4**

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson Education.

Module	Content	Content Mapped to CO No	Hours
I	**Part IV** Sections 24 : Non commutative examples **Part V** Section 26 : Homomorphisms and factor rings Section 27 : Prime and Maximal Ideals **Part VI** Section 29 : Introduction to Extension fields	4 2,5 2,4 1	(20 hours)
II	**Part VI** Section 31 : (31.1 – 31.18)Algebraic extensions Section 32 : Geometric constructions Section 33 :Finite fields.	**1,2** **3** **1,4**	(20 hours)
III	**Part X** Section 48 : Automorphisms and fields Section 49 : (49.1 -49.5) The isomorphism extension theorem (proof of the theorem excluded) Section 50 : Splitting fields	**2** **1,2** **1,5**	(25 hours)

IV	**Part X** Section 51 : Separable extensions Section 53 : Galois theory Section 54 : Illustrations Of Galois Theory Section 55 : (55.1-55.6)Cyclotomic Extensions	1,5 5 4,5	(25 hours)

References

[1] I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi,1975.

[2] Hungerford, Algebra,Springer

[3] M. Artin, Algebra, Prentice -Hall of India, 1991

[4] N. Jacobson, Basic Algebra Vol. I, Hindustan PublishingCorporation

Question Paper Pattern

.	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	2	2	1
Module III	3	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Second Semester
Programme - M.Sc. Mathematics
PG2MATC09 – ADVANCED ABSTRACT ALGEBRA

Time: Three Hours Maximum Weight: 30

Part A

Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1 State Fundamental Homomorphism Theorem.

2 If R is a ring with unity, and N is an ideal of R containing a unit, then prove that N = R.

3 Explain prime ideal with an example.

4 Verify whether the polynomial $8x^3+6x^2-9x+24$ is irreducible over Q.

5 Find degree and a basis for $Q(\sqrt{2},\sqrt{6})$ over $Q(\sqrt{3})$?

6 Find the splitting field of $\{x^2 -2, x^2 -3\}$ over Q.

7 Define index of E over F with an example.

8 Find all conjugates in C $\sqrt{1+\sqrt{2}}$ over $Q(\sqrt{2})$

9 Define separable extension with an example.

10 State Primitive Element Theorem.

(8x1=8 Weights)

Part B

Short Essay Questions

*(Answer any **six** questions. Each question carries Weight **2**)*

11 Let R be a commutative ring with unity, then prove that M is a maximal ideal if and only if R/M is a field

12 If F is a field then prove that every ideal in F[x] is principal

13 State and Prove Fundamental theorem of Algebra..

14 Trisecting the angle is impossible.Justify.

15 Explain fixed field with an example.

16 If E $\leq \bar{F}$ is a splitting field over F, then prove that every irreducible polynomial in F[x] having a zero in E splits in E.

17 Prove that everyfinite field is perfect.

18 Explain the Galois group of a cyclotomic extension

(6 x 2 = 12 Weights)

Part C

Long Essay Questions

*(Answer any **two** questions.Each question carries Weight **5**)*

19 State and Prove Kronecker's theorem

20 (a) Let E be a finite extension field of a field F and let K be a finite extension field of E. Prove that K is a finite extension field of F and [K: F]=[K: E][E: F].

(b) A finite field GF(p^n) of p^n elements exists for every prime power p^n.

21 State and prove Conjugation Isomorphisms theorem.

22 Explain Main theorem of Galois Theory.

(2x5=10 Weights)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
SECOND SEMESTER
PG2MATC10 –NUMERICAL ANALYSIS WITH PYTHON

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Develop programming skills in core Python.	C	7
2	Write Python programs to find mathematical solutions to numerical problems	Ap	2,4
3	Locate and use information to numerically solve problems	An	2,3,4
4	Think critically by analyzing practical problem and understanding the mathematical basis of the problem	R,E	5,6
5	Develop and implementing algorithms for solving application problems	C	3,4,5
6	Create Graph with python tools and solves problems using this idea	C	4

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text :**[1] Amit Saha,** Doing Math with Python, No Starch Press, 2015.

[2] Jaan Kiusalaas, Numerical Methods in Engineering with Python3, Cambridge University Press.

Though any distribution of Python 3 software can be used for practical sessions, to avoid difficulty in getting and installing required modules like numpy, scipy, etc, and for uniformity, the Python3 package *Anaconda 2018.x* (https://www.anaconda.com/distribution/#download-section) may be installed and used for the practical sessions. However, a brief introduction on how to use Python IDLE 3 also should be given.

Module	Contents	Contents Mapped to CO No	Hours
I	**Text 2 :** **Chapter 1 : Introduction to Python** 1.1 : General Information 1.2 : Core Python	 1 1	

	1.3 : Functions and Modules 1.4 : Mathematics Modules 1.5 : numarray Module For a detailed review it is recommended to consider chapters 2, 3, 4,5,6, 7 8 and 9 of the Reference text [1] **Text 1 :** **Chapter 1**Section : Complex Numbers **Chapter 2** Working with Lists and Tuples Creating Graphs with Matplotlib Plotting with Formulas	1,2 1,2 1,2,3 1 1,2 1,2,6 1,2,6	(20 hours)
II	**Text 1 :** **Chapter 4: Algebra and Symbolic Math with SumPy** Defining Symbols and Symbolic Operations Working with Expressions Solving Equations Plotting Using SymPy, Problems on factor finder Summing a series Solving single variable inequalities. **Chapter 7: Solving Calculus Problems** Assumptions in SymPy Finding the Limit of Functions Finding the Derivative of Functions Finding the Integrals of Functions	 1,2 1,2 1,2 1,2,6 1,2,3 1,2,3,4 1,2 1,2 1,2,3,4 1,2,4 1,2,4	(22 hours)
III	**Text 2 :** **Chapter 3: Interpolation and Curve Fitting** 3.1 Introduction 3.2 Polynomial Interpolation - Lagrange'sMethod, Newton's Method and Limitations of Polynomial Interpolation (Neville’s Method omitted) **Chapter 4 : Roots of Equations** 4.1 Introduction 4.3 Method of Bisection 4.5 Newton-Raphson Method.	 1,2 1,2,3,5 1,2 1,2,4,5 1,2,4,5	(24 hours)
IV	**Text 2 : Chapter 2 : Systems of Linear Algebraic Equations** 2.1 Introduction 2.2 Gauss Elimination Method (excluding Multiple Sets of Equations), 2.3 LU Decomposition Methods (Doolittle's Decomposition Method only) **Chapter 6 : Numerical Integration** 6.1 Introduction 6.2 Newton-Cotes Formulas, Trapezoidal rule, Simpson's rule and Simpson's 3/8 rule	 1,2 1,2,4,5 1,2,4,5 1,2 1,2,34,5	(24 hours)

(1) Instead of assignments, a practical record book should be maintained by the students. Atleast 15 programmes should be included in this record book.
(2) Internal assessment examinations should be conducted as practical lab examinations by the faculty handling the paper.
(3) End semester examination should focus on questions including definitions, concepts and methods discussed in the each units and not for writing long programs. However, more importance should be given to theory in the end semester examinations as internal examinations will be giving more focus on programming sessions.

References

[1] Jason R Briggs, Python for Kids-APlayful Introduction to Programming, No Starch Press

[2] A primer on scientific programming with python, 3rd edition, Hans PetterLangtangen, Springer

[3] Vernon L. Ceder, The Quick Python Book, Second Edition, Manning.

[4] NumPy Reference Release 1.12.0, Written by the NumPy community. (available for free download at https://docs.scipy.org/doc/numpy-dev/numpy-ref.pdf)

[5] S. D. Conte and Carl de Boor, Elementary Numerical Analysis – An algorithmic approach, Third Edition, McGraw-Hill Book Company.

[6] S. S. Sastry, Introductory Methods of Numerical Analysis, Fifth Edition, PHI.

[7] A Byte of Python, Swaroop C H

[8] Numerical Methods, E Balagurusamy, Tata McGraw-Hill Publishing Company Limited, New Delhi.

[9] Ajith Kumar B.P. Python for Education

(For more problems visit
https://www.nostarch.com/doingmathwithpython/,
https://doingmathwithpython.github.io/author/amit-saha.html and
https://projecteuler.net/.)

Question paper pattern

	Part A Weight 1 Short questions	Part B Weight 2 Short essays	Part C Weight 5 Long essays
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Second Semester
Programme : M.Sc. Mathematics
PG2MATC10 : NUMERICAL ANALYSIS WITH PYTHON

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Write a note on tuples.
2. Explain 'while' statement in python program with an example.
3. Write a Python program to find the mean of three numbers.
4. What will be the output of print str[2:5] if str='hello world!' in python program?
5. Explain the use of functions factor() and expand() in python program.
6. Write a program to find the definite integral $\int_0^1 kx\,dx$, where k is a constant.
7. Define interpolation.
8. Find the root of the equation $f(x) = x^2 - 3x + 2$ in the vicinity of $x = 0$ using Newton-Raphson's method.
9. Write a short note on a system of algebraic equations.
10. Write a note on LU decomposition of matrices.

($8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Write a Python program that list prime numbers up to a given number n.
12. Write a Python program to draw a graph showing a line passing through the points $(1, 2)$, $(2, 4)$, and $(3, 6)$.

Turn Over

13. Write a Python program to print the series $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots + \frac{x^n}{n}$.

14. Write a Python program to find the derivate of the function $y = 5x^2 + 3x$ with respect to x.

15. Write a note on Lagrange's method for polynomial interpolation.

16. What are the limitations for polynomial interpolation.

17. Derive Newton-Cotes Formulas.

18. Derive Simpsons 1/3 rule from Newton-Cotes formulas.

$(6 \times 2 = 12$ weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. (i) Formulate a Python program with an example to find out all the values in the list that are greater than the specified number.

    (ii) Write a Python program to input the expression $3x^2 + 2x + 5$, $x^3 + 7x$, calculate the product and display them.

20. (i) Write a Python program to solve the polynomial inequality $x^2 + 4 < 0$.

    (ii) Write a Python program to find the derivative of the function $S(t) = 5t^2 + 2t + 8$ with respect to t using first principle.

21. (i) Given the data points

x	0	2	3
y	7	11	28

    use Lagrange's method to determine y at $x = 1$. Also write its algorithm.

    (ii) Find all the zeros of $f(x) = x \tan x$ in the interval $(0, 20)$ by the method of bisection. Utilize the functions rootsearch and bisect.

22. (i) Write a Python program to find the solution of a 3×3 system using the basic Gauss elimination method.

    (ii) Write an algorithm and corresponding Python program of finding the integral using Trapezoidal Rule and hence evaluate $\int_a^b (x^3 + 1)\, dx$.

$(2 \times 5 = 10$ weight)

SEMESTER III

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
THIRD SEMESTER
PG3MATC11 – SPECTRAL THEORY

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome
Upon successful completion of the course student will be able to

CO No	CourseOutcome	Cognitive Level	CO Mapped to PSO
1	Understand the concepts of convergence and spectral theory	U	1,6
2	Analyze and identify operators	R,An	1,2,4,6
3	Perform computations	E	3,5,7
4	Apply theorems	Ap	2,3,4,5
5	Invent new development	C	1,4

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications, John Wiley and sons, New York

Module	Contents	Contents mapped to CO	Hours
I	**Chapter 4.Fundamental Theorems for Normed and Banach Spaces** 4.8 Strong and Weak Convergence 4.9 Convergence of Sequence of Operators and Functionals 4.12 Open Mapping Theorem 4.13 Closed Linear Operators, Closed Graph Theorem **Chapter 5. Further Applications : Banach fixed point theorem** 5.1 Banach fixed point theorem	1,4 1,2 4 4 4	(25hrs)
II	**Chapter 7.Spectral theoryof Linear Operators in Normed Spaces** 7.1 Spectral Theory in Finite Dimensional Normed Space 7.2 Basic Concepts	1,5 1	(25hrs)

	7.3 Spectral Properties of bounded linear operators 7.4 Further properties of Resolvant and Spectrum 7.5 Use of Complex Analysis in Spectral Theory 7.6 Banach algebras 7.7 Further properties of Banach Algebras	2,4 1,2,4 3 5 4,5	
III	**Chapter 8.Compact Linear Operators on Normed Space and their Spectrum** 8.1 Compact linear operators on normed spaces 8.2 Further properties of compact linear operators 8.3 Spectral properties of compact linear operators on normed spaces 8.4 Further spectral properties of compact linear operators	2,5 5 3,4 5	(20hrs)
IV	**Chapter 9.Spectral Properties of Bounded Self AdjointLinear Operators** 9.1 Spectral Properties of Bounded Self-Adjoint Linear operators 9.2 Further Spectral Properties of Bounded Self-adjoint linear operators 9.3 Positive Operators 9.5 Projection Operators 9.6 Further Properties of Projections	1,5 2,3 2 2,5 4,5	(20hrs)

References

[1] Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York1963.

[2] Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi1989

[3] Somasundaram. D, Functional Analysis, S.ViswanathanPvt Ltd, Madras,1994

[4] Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut:1995-96

Question paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short questions	Short essays	Longessays
Module I	3	2	1
Module II	2	2	1
ModuleIII	3	2	1
ModuleIV	2	2	1
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Third Semester
Programme - M.Sc. Mathematics
PG3MATC11 – SPECTRAL THEORY

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Let $T_n \in B(X, Y)$ where X is a Banach space and Y a normed space. If (T_n) is strongly operator convergent with limit T, then show that $T \in B(X, Y)$.
2. Show that the null space $\mathcal{N}(T)$ of a closed linear operator $T: X \to Y$ is a closed subspace of X.
3. Show that the eigen values of a skew Hermitian matrix are purely imaginary or zero.
4. Define spectral radius of an operator $T \in B(X, X)$ and find an upper bound for it.
5. If $x \in A$ is invertible and commutes with $y \in A$, then show that x^{-1} and y also commute, where A is a Banach algebra.
6. Show that a compact linear operator is continuous, whereas the converse is not generally true.
7. State the sequential criterion for compactness of a linear operator $T: X \to Y$ where X and Y are normed spaces.
8. Define a totally bounded subset of a metric space.
9. Show that all the eigen values of a bounded self-adjoint linear operator on a complex Hilbert space H (if they exist) are real.
10. For any projection P on a Hilbert space H, show that $P \geq 0$

(8 x 1 = 8 Weights)

Part B
Short Essay Questions

*(Answer any **six** questions. Each question carries Weight**2**)*

11. If $x_n \xrightarrow{w} x$ in a normed space X show that the sequence $(\|x_n\|)$ is bounded.
12. Show that in X' weak convergence implies weak* convergence. What about the converse of this statement?

13. Prove that all matrices representing a given linear operator $T: X \longrightarrow X$ on a finite dimensional normed space X relative to various bases for X have the same eigen values.
14. Show that the set of eigen vectors corresponding to different eigen values of a linear operator T on a vector space X is linearly independent.
15. Define a Banach algebra. Show that the space $B(X, X)$ of all bounded linear operators on a complex Banach space $X \neq \{0\}$ is a Banach algebra with identity.
16. Let $T: X \longrightarrow X$ be a compact linear operator on a Banach space X. Prove that every spectral value $\lambda \neq 0$ of T (if it exists) is an eigen value of T.
17. Prove that the spectrum of a bounded self adjoint linear operator $T : H \longrightarrow H$ on a complex Hilbert space H is real.
18. Define a positive operator. Prove that sum of two positive operators A and B is positive. Show by an example that their composition AB may not be positive in general.

(6 x 2 = 12 Weights)

Part C
Long Essay Questions

(Answer any two questions. Each question carries weight 5)

19. State and prove Banach fixed point theorem. Illustrate with an example that completeness is essential in the theorem.
20. Let A be a complex Banach algebra with identity e. Then for any $x \in A$ prove that $\sigma(x)$ is compact and nonempty.
21. Prove that the set of eigen values of a compact linear operator on a normed space is countable and the only possible point of accumulation is $\lambda = 0$.
22. For any bounded self adjoint linear operator on a complex Hilbert space H, prove that
$\|T\| = \sup_{\|x\|=1} |\langle Tx, x\rangle|$.

(2 x 5 = 10 Weghts)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
THIRD SEMESTER
PG3MATC12 – COMPLEX ANALYSIS

(5 hours/week) **(Credit : 4)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No.	Course Outcome	Cognitive Level	CO mapped to PSO No
1	Understand concept of representation of complex numbers in the extended complex plane.	U	1
2	Explain the concept of (complex) differentiation and integration of functions defined on the complex plane and their properties.	R,U	1,4,6
3	Represent analytic functions as power series	Ap	1,2,4,6
4	Apply various theorems and results to solve problems.	Ap	1,2,6
5	Identify zeros and classify singularities of complex functions	E,U	1,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : **John B. Conway**, Functions of One Complex Variable, Second Edition.

Module	Contents	Content mapped to CO No	Hours
I	**I. The Complex Number System** 6. The extended plane and its spherical representation **III. Analytic functions** 1. Power series 2. Analytic functions 3. Analytic functions as mappings, Mobius transformations	1 3,4 3,4 4	(25 hrs.)
II	**IV. Complex Integration** 2. Power series representation of analytic functions 3. Zeros of an analytic function 4. The index of a closed curve, 5. Cauchy's Theorem and Integral Formula 6. The Homotopic version of Cauchy's Theorem and simple connectivity	3,4 4,5 5 2,4 2,4	(25 hrs.)

III	**IV. Complex Integration** 7. Counting zeros, the Open Mapping Theorem 8. Goursat's Theorem **V. Singularities** 1. Classification of singularities 2. Residues 3. The Argument Principle	4,5 4 5 5 4,5	(20 hrs)
IV	**VI. The Maximum Modulus Theorem** 1. The Maximum Principle, 2. Schwarz's Lemma, 3. Convex Functions and Hadamard's Three circles Theorem	 4 2,4 4	(20 hrs)

References

[1] **H. Cartan**: Elementary Theory of analytic functions of one or several variables; Addison - Wesley Pub. Co.; 1973

[2] **B. Chaudhary**, The elements of Complex Analysis, WileyEastern

[3] **T.W. Gamelin**: Complex Analysis; Springer-Verlag, NY Inc.; 2001

[4] **S. Lang**, Complex Analysis, Springer

[5] T.O. Moore and E.H. Hadlock: Complex Analysis, Series in Pure Mathematics-Vol. 9; World Scientific; 1991

[6] **L. Pennisi**: Elements of Complex Variables (2nd Edn.); Holf, Rinehart & Winston 1976

[7] **H.A. Priestly**, Introduction to Complex Analysis, Clarendon press, Oxford, 1990

[8] **R. Remmert**: Theory of Complex Functions; UTM , Springer-Verlag, NY; 1991

[9] **W. Rudin**: Real and Complex Analysis (3rd Edn.); Mc Graw - Hill InternationalEditions; 1987

[10] **H. Sliverman**: Complex Variables; Houghton Mifflin Co. Boston; 1975.

Question Paper Pattern

.	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Third Semester
Programme - M.Sc. Mathematics
PG3MATC12 - COMPLEX ANALYSIS

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions/Problems
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Find the radius of convergence of $\sum_{n=0}^{\infty} a^n z^n$, *a* in C.
2. What is meant by Mobius transformation? Define translation and rotation.
3. State Orientation Principle. Find the image of the line z = c under $w = z^2$.
4. What is Cauchy's Integral Formulae? Evaluate $\int_{|z|=1} \frac{z^2+2}{z-3}\, dz$
5. Define winding number of a closed curve. Evaluate $\int_{|z|=1} \frac{1}{x-2} dz$
6. Let G be an open set which is a – star shaped. If Υ_0 is the curve which is constantly equal to *a* then prove that every closed rectifiable curve in G is homotopic to Υ_0 .
7. Define isolated singularity and pole. Give an example for each.
8. Define limit superior and limit inferior of a function *f* (z).
9. Define a convex function with example.
10. What is meant by meromorphic function. Give an example.

(8 x 1 = 8 Weights)

Part B
Short Essay Questions/Problems
*(Answer any **six** questions. Each question carries Weight **2**)*

11. If G is open and connected and f : G → D is differentiable with f '(z) = 0 for all z in G, then prove that f is constant.
12. If S is a Mobius transformation then show that S is the composition of translations, dilations, and the invertion.
13. Let G be a region and let f : G → D be a continuous function such that $\int_r f = 0$ for every triangular path T in G, then prove that f is analytic in G.
14. Explain with proof the Argument Principle.
15. State and prove Rouche's Theorem.
16. State and prove Liouvillies theorem..

Turn Over

17. Let f : D → D be a one-one analytic map of D onto itself and suppose $f(a) = 0$. Then prove that there is a complex number c with $|c| = 1$ such that $f = c\varphi_\alpha$

18. Show that a function $f : [a , b] \to \mathbf{R}$ is convex if and only if the set

$$A = \{(x,y) : a \le x \le b \text{ and } f(x) \le y\} \text{ is convex .}$$

(6 x 2 = 12 Weights)

Part C

Long Essay Questions/Problems

(*Answer any two questions. Each question carries Weight 5)*

19. Let u and v be real valued functions on a region G and suppose that u and v have continuous partial derivatives. Then prove that $f : G \to C$ defined by $f(z) = u(z) + iv(z)$ is analytic if and only if u and v satisfy the Cauchy – Riemann equations.
20. Sate and prove Cauchy's theorem third version.
21. State residue theorem and hence show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$
22. State and prove all three versions of Maximum Modulus Theorem

(2x5=10 Weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
THIRD SEMESTER
PG3MATC13-ANALYTIC NUMBER THEORY

(5 hours/ week) **(Credit:4)**
(Total Hours:90) (Maximum Weight :30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Explain the properties of prime number theory	U	4,5
2	Understand the arithmetic functions and their utility in the prime number theory	U	1,4
3	Identify the general method of analysis to obtain the results about integers and prime numbers	R,U	1,2,4
4	Apply these prime number theory and techniques in various applications in applied mathematics	Ap	1,4,6
5	Execute calculations and derive different identities in number theory.	E,C	1,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text: Tom M.Apostol- Introduction to Analytic Number Theory, Springer International, Student Edition, Narosa Publishing House

Module	Content	Content Mapped to CO No	Hour
I	**Arithmetic Functions Dirichlet's Multiplication and averages of arithmetical functions**		30 Hrs
	2.1 Introduction	1	
	2.2 Mobius Function μ(n)	1,2	
	2.3 The Euler Totient Function φ(n)	1,2	
	2.4 A relation connecting φ and μ	1,2,3	
	2.5 A product formula for φ(n)	2	
	2.6 The Dirichlet's product of arithmetical functions	1,2	
	2.7 Dirichlet Inverses and Mobius inversion formula	1,2	
	2.8 The Mangoldt Function Λ(n)	1,2	
	2.9 Mutiplicative functions	2	
	2.10 Mutiplicative functions and Dirichlet's multiplication	2	
	2.11 The inverse of completely multiplicative function	1,2	
	2.12 Lioville's function λ(n)	1,2	

	2.13 The divisor functions σ_α (n) 2.14 Generalized convolutions 2.15 Formal power series 2.16 The Bell series of an arithmetical function 2.17 Bell series and Dirichlet multiplication 3.1 Introduction 3.2 The Big oh notation and asymptotic equality of the functions 3.3 Euler's summation formula 3.4 Some elementary asymptotic formulas 3.5 The average order of d(n) 3.6 The average order of the divisor functions $\sigma_\alpha(n)$ 3.7 The average order of φ(n) 3.8 An application of distribution of lattice points visible from the origin 3.9 The average order of μ(n) and of Λ(n) 3.10 The partial sum of a Dirichlet product 3.11 Applications to μ(n) and Λ(n)	1,2 1,2,3 3,4,5 2 2,4,5 1,2 2 4,5 4,5 1,2 2 2 5 1,2 2,3 3,4,5	
II	**Some Elemetary theorems on the distribution of prime numbers** 4.1 Introduction 4.2 Chebyshev's functions ψ(x) and ϑ(x) 4.3 Relations connecting ϑ(x) and π(x) 4.4 Some equivalent forms of prime number theorem 4.5 Inequalities for π(n) and p_n 4.6 Shapiro's Tauberian theorem 4.7 Applications of Shapiro's theorem 4.8 An asymptotic formula for the partial sums $\sum_{p\le x}(1/p)$	1 1,2 1,2,4,5 3,4,5 4,5 1,2,3 4,5 3,4,5	20 Hrs
III	**Congruences** 5.1Definition and basic properties of congruences 5.2 Residue classes and complete residue systems 5.3 Linear congruences 5.4 Reduced residue systems and the Euler – Fermat theorem 5.5 Polynomial congruences modulo p, Lagrange's theorem 5.6 Applications of Lagrange's theorem 5.7 Simultaneous linear congruences. The Chinese reminder theorem 5.8 of Chinese reminder theorem	1,2 1,2 1,2 1,2,3,4 1,2,3,4 3,4,5 4,5 4,5	25 Hrs

	5.9 Polynomial congruences with prime power moduli	3	
IV	**Primitive roots and partitions** 10.1The exponent of a number mod m. Primitive roots 10.2 Primitive roots and reduced residue systems 10.3 The nonexistence of Primitive roots mod 2^{α} for $\alpha \geq 3$ 10.4 The existence of Primitive roots mod p for odd primes p 10.5 Primitive roots and quadratic residues. 14.1 Introduction 14.2 Geometric representation of partitions 14.3 Generating functions for partitions 14.4 Euler's pentagonal-number theorem	 1,2,3 2,3 3 3 3,4 1,2 1,2,3,4 3,4,5 3,4,5	15 Hrs

Reference

[1] Hardy G.H and Wright E.M , Introduction to the Theory of numbers, Oxford, 1981

[2] Leveque W.J, Topics in Number Theory, Addison Wesley, 1961.

[3] J.P Serre, A Course in Arithmetic, GTM Vol. 7, Springer-Verlag, 1973

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	2	2	1
Module III	3	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code **Reg.No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Third Semester
Programme - M.Sc. Mathematics
PG3MATC13- ANALYTIC NUMBER THEORY

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Define Dirichlet convolution. Show that Dirichlet multiplication is commutative and associative.
2. Prove that d(n) is odd iff n is a square.
3. Assume f is a multiplicative function. Prove that $f^{-1}(n) = \mu(n)$ for every square free n.
4. If a > 0 and b > 0, then $\pi(ax)/\pi(bx)$ as $x \to \infty$.
5. Define Chebychev's ψ function and Chebychev's ϑ function. Show that

$$\lim_{x\to\infty}\left(\frac{\psi(x)}{x} - \frac{\vartheta(x)}{x}\right) = 0.$$

6. State and prove converse of Wilson's theorem.
7. Assume (a,m)=1. Then prove that the linear congruence $ax \equiv b(mod\ m)$ has exactly one solution.
8. Solve the linear congruence $5x \equiv 12(mod\ 23)$.
9. Given $m \geq 1$. $(a, m) = 1$. Let $f = exp_m(a)$. Then prove that $a^k \equiv a^h(mod\ m)$ iff $k \equiv h(modf)$.
10. State Euler's pentagonal number theorem.

(8x1=8 Weight)

Part B
Short Essay Questions
*(Answer any **six** questions. Each question carries Weight **2**)*

11. State and Prove Euler's summation formula.
12. Find the average order of d(n).
13. State and prove Abel's identity.
14. Let P_n denote the nth prime. The show that the following asymptotic relations are logically equivalent:

    $a)\ \lim_{x\to\infty}\frac{\pi(x)logx}{x} = 1$, $b)\ \lim_{x\to\infty}\frac{\pi(x)log\pi(x)}{x} = 1$, $c)\ \lim_{n\to\infty}\frac{P_n}{nlogn} = 1.$
15. For any prime p all the coefficients of the polynomial $f(x) = (x-1)(x-2)\ldots(x-p+1) - x^{p-1} + 1$ are divisible by p.
16. State and prove Euler Fermat Theorem.

Turn Over

17. Let p be an odd prime and d be any positive divisor of p-1. Then in every reduced residue system mod p there are exactly $\varphi(d)$ numbers a such that $exp_p(a) = d$.

18. Define exponent of a modulo m and primitive root modulo m .Given(a.m)=1, let $f = exp_m(a)$. Then show that $exp_a(a^k) = \frac{exp_m(a)}{(k.f)}$.

(6x2=12 Weight)

Part C

Long Essay Questions

*(Answer any **two** questions. Each question carries Weight **5**)*

19. a) Define multiplicative functions. Give an example. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative

 b) Establish the relation connecting Euler's φ function and Mobius function μ.

 c) State and Prove Generalised inversion formula.

20. State and prove Shapiro's Tauberian theorem

21. a) State and Prove Lagrange's theorem.

 b) State and prove Wolstenholme's theorem

22. For $|x| < 1$, prove that $\prod_{m=1}^{\infty} \frac{1}{1-x^m} = \sum_{n=0}^{\infty} p(n)x^n$, where $p(0) = 1$.

(2x5=10 Weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
THIRD SEMESTER
PG3MATC14 - Optimization Techniques

(5 hours/week) **(Credit : 4)**

(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand classical optimization techniques and numerical methods of optimization.	U	1,6
2	Enumerate fundamentals of Integer programming technique and apply different techniques to solve various optimization problems arising from engineering areas.	E,Ap	1,3,4,5
3	Analyze decision making problems and solve them using the techniques acquired from game theory.	An	1,2,4,5
4	Solve non linear programming problems.	Ap	2,4
5	Solve the problems in network analysis like minimum path problem, sequencing job problem, maximum flow problem etc.	Ap	2,3,4,5

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text - 1 K.V. Mital and C. Mohan, Optimization Methods in Operation Research And Systems Analysis, 3rd edition.

Text - 2 Ravindran, Philips and Solberg. Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.

Module	Contents	Content Mapped to CO	Hours
I	**Text 1** **Chapter 6: INTEGER PROGRAMMING** 1. Introduction 2. ILP in two dimensional space 3. General ILP and MILP problems 4. Examples of Section 2 continued 5. Cutting planes 6. Examples 7. Remarks on cutting plane methods 8. Branch and bound method – examples 9. Branch and bound method - general description 10. The 0 – 1 variable.	1,2	15

<table>
<tr><td>II</td><td>Text 1
Chapter 5 :FLOW AND POTENTIALS IN NETWORKS

1. Introduction
2. Graphs-definitions and notations
3. Minimum path problem
4. Spanning tree of minimum length
5. Problem of minimum potential difference
6. Scheduling of sequential activities
7. Maximum flow problem
8. Duality in the Maximum flow problem
9. Generalized problem of maximum flow.

Chapter 7:ADDITIONAL TOPICS IN LINEAR PROGRAMMING

1. Introduction
2. Sensitivity analysis
3. Changes in b_i
4. Changes in c_j
5. Changes in a_{ij}
6. Introduction of new variables
7. Introduction of new constraints
8. Deletion of variables
9. Deletion of constraints
15. Goal programming</td><td>1,5</td><td>25</td></tr>
<tr><td>III</td><td>Text 1
Chapter 12 : THEORY OF GAMES

1. Introduction
2. Matrix (or rectangular) games
3. Problem of games
4. Minimax theorem, saddle point
5. Strategies and pay off
6. Theorems of matrix games
7. Graphical solution
8. Notion of dominance
9. Rectangular game as an LP problem</td><td>1,3</td><td>20</td></tr>
</table>

IV	**Text 2** **Chapter 11 : NON- LINEAR PROGRAMMING** **I. Basic concepts** 11.1 Introduction 11.2 Taylor's series expansion **II. Unconstraint optimization** 11.3 Fibonacci Search and golden section search 11.4 Hooke and Jeeves search algorithm 11.5 Gradient projection search III. **Constraint optimization problems:Equality constrainrs** 11.6 Lagrange multipliers 11.7 Equality constraint optimization: constrained derivatives 11.8 Projected gradient methods with equality constraints **IV. Constraint optimization problems: Inequality constraints** 11.9 Non-linear optimization: Kuhn-Tucker conditions 11.10 Quadratic Programming 11.11 Complementary Pivot algorithms.	4	30

References

[1] **S.S. Rao**, Optimization Theory and Applications, 2nd edition, New Age International Pvt.

[2] **J.K. Sharma**, Operations Research: Theory and Applications, Third edition, Macmillan India Ltd.

[3] **Hamdy A. Thaha,** Operations Research – An Introduction, 6th edition, Prentice Hall of India Pvt. Ltd.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code (Pages 2) **Reg. No.**

. **Name :**

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Third Semester
Programme : M.Sc. Mathematics
PG3MATC14 - Optimization Techniques

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

(Answer any ***eight*** *questions. Each question carries Weight* ***1****)*

1. What is the relation between T_F, S_F and $[T_F]$, that is the set of feasible solution of ILPP/ MILPP, related LP and convex hull of $[TF]$? Justify .
2. Explain the term Branching.
3. Define Arborescence with a diagram.
4. Distinguish between a) Path and Chain b) Cycle and Circuit.
5. For any feasible flow $\{x_i\}, i = 1, 2, \ldots m$ in a graph, prove that the flow x_0 in the return arc is not greater than the capacity of any cut in the graph.
6. Explain the sensitivity analysis if a new constraint is added to the linear programming problem.
7. State the fundamental theorem of rectangular games.
8. Define rectangular games and pay off matrix.
9. Explain unimodal function and multimodal function in non-linear programming problem with diagrams.
10. Define quadratic programming problem.

($8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems

(Answer any ***six*** *questions. Each question carries Weight* ***2****)*

11. Prove that for an ILP or MILP its optimal solution $X \in T_F$ (set of feasible solutions), is also an optimal solution of $X \in [T_F]$ (convex hull of T_F). Also prove its converse.

Turn Over

12. Explain cutting plane algorithm for Integer programming problem.

13. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.

14. Describe the minimum path problem.

15. State and prove the minimax theorem of the saddle point (X_0, Y_0).

16. Prove that $E(X, Y_0) \leq E(X_0, Y_0) \leq E(X_0, Y)$ is equivalent to $E(\zeta_i, Y_0) \leq E(X_0, Y_0) \leq E(X_0, y_j)$ where ζ_i, η_j, $i = 1, 2 \ldots m$ and $j = 1, 2....n$ are pure strategies.

17. Define Hessian Matrix. Write the Hessian matrix of the function
$$f(x) = {x_1}^2 + {x_2}^2 - 2x_1 + x_1x_2 + 1 .$$

18. Describe the complementary pivot algorithm.

(6 × 2 = 12 weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. Use Branch and Bound method to solve
$Maximise\ Z = 3x_1 + 4x_2$ Subject to
$2x_1 + 4x_2 \leq 13$
$-2x_1 + x_2 \leq 2$
$2x_1 + 2x_2 \geq 1$
$6x_1 - 4x_2 \leq 15$, where $x_1, x_2 \geq 0$ and are integers.

20. What do you mean by sensitivity analysis? Discuss the changes in the value of a_{ij}.

21. State and prove the fundamental theorem of Rectangular games.

22. Using Newtons method Minimize $f(x) = 2(x_1 + x_2)^2 + 2({x_1}^2 + {x_2}^2)$ with initial point $(5, 2)$.

(2 × 5 = 10 weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
THIRD SEMESTER
PG3MATC15 – PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

(5 Hours/week) **(Credit 4)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Familiarize the geometric objects, Identifies and classifies various types of partial differential equations	R,U	1,6
2	Solve PDE and analyze the solution to get information about the parameters involved in the model.	Ap,An	2,3,4
3	Represent solutions of three important classes of PDE explicitly - Heat equations Laplace equation and wave equation for initial value problems.	An	2,3,4
4	Solve Integral equations.	Ap	2,4,5
5	Relate Integral and differential Equations.	An	6
6	Explain partial and integral equations	U	1,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text 1 : Amaranath T.: Partial Differential Equations, Narosa, New Delhi, 1997.

Text 2 : Hildbrand. F.B, Methods of Applied Mathematics- (PHI.1972, II nd Edition).

Module	Contents	Content Mapped to CO No	Hours
I	**TEXT 1** **Chapter 1 : First Order P.D.E.** 1.1 : Curves and surfaces 1.2 : Genesis of First Order P.D.E 1.3 : Classification of Integrals 1.4 : Linear Equations of the First Order 1.5 : Pfaffian Differential Equations 1.6 : Compatible Systems 1.7 : Charpit's Method.	 1 1,6 1,6 1,2,6 1,2 1,2 1,2	(25 hrs)

II	**TEXT 1** **Chapter 1 : First Order PDE (continued)** 1.9 : Integral Surfaces Through a Given Curve 1.10 : Quasi-Linear equations 1.11 : No-linear first order P.D.E (Omit proof of Lemma 1.11.1) **Chapter 2 : Second Order PDE** 2.1 : Genesis of Second Order P.D.E 2.2 : Classification of Second Order P.D.E 2.3 : One Dimensional Wave Equation 2.3.1 : Vibrations of an Infinite String 2.3.2 : Vibrations of a Semi-Infinite String 2.3.3 : Vibrations of String of Finite Length.	 1,2 1,2 1,2 1,3,6 1,3,6 1,3 1,3 1,3 1,3	(20 hrs)
III	**TEXT 1** **Chapter 2 : Second Order PDE(continued)** 2.4.1 : Boundary Value Problems 2.4.2 : Maximum and Minimum Principles 2.4.3 : The Cauchy Problem 2.4.4 : The Dirichlet Problem for Upper Half Plane 2.4.5 : The Neumann Problem for the Upper Half Plane 2.4.6 : The Dirichlet Problem for a Circle 2.4.7 : The Dirichlet Exterior Problem for a Circle 2.4.8 : The Neumann Problem for a Circle 2.4.9 : The Dirchlet Problem for a Rectangle 2.4.10 :Harnack's Theorem 2.5.1 : Heat Conduction –Infinite Rod Case 2.5.2 : Heat conduction – Finite Rod Case (upto exercise 2.5.1)	 1,3 1,3 1,3 1,3 1,3 1,3 1,3 1,3 1,3 1,3 1,3 1,3	(25 hrs)
IV	**TEXT 2** **Chapter 3 : Integral Equations** 3.1 : Introduction 3.2 : Relation between differential and integral equations 3.3 : The Green's function 3.6 : Fredholm equations with separable kernels 3.7 : Illustrative examples 3.8 : Hilbert-Schmidt theory 3.9 : Iterative methods for solving equations of the second kind 3.10 : The Neumann series	 4,6 4,5,6 4,5 4,5 4,5 4,5 4,5 4,5	(20 hrs)

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References

[1] **A. Chakrabarti**: Elements of Ordinary Differential Equations and special functions; Wiley Eastern Ltd, New Delhi; 1990

[2] **E.A. Coddington**: An Introduction to Ordinary Differential Equations Printice Hall of India, New Delhi; 1974.

[3] **R. Courant and D. Hilbert**: Methods of Mathematical Physics-Vol I; Wiley Eastern Reprint; 1975.

[4] **P. Hartman**: Ordinary Differential Equations; John Wiley & Sons; 1964

[5] **F. John**: Partial Differential Equations; Narosa Pub House New Delhi; 1986

[6] **Phoolan Prasad, Renuka Ravindran**: Partial Differential Equations; Wiley Eastern Ltd, New Delhi; 1985.

[7] **L.S. Pontriyagin**: A Course in Ordinary Differential Equations; Hindustan Pub. Corporation, Delhi; 1967.

[8] **M. D. Raisinghania**, Integral Equations and Boundary Value Problems 10e, S. Chand Publishing.

[9] **I. N. Sneddon**: Elements of Partial Differential Equations; McGraw-Hill International Edn.; 1975.

[10] **Yehuda Pinchover and Jacob Rubinstein**, An Introduction to Partial Differential Equations, Cambridge University Press.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	2	2	1
Module III	3	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code . (Pages 2) **Reg. No.**
Name.

MODEL QUESTION PAPER
M.Sc. DEGREE(C.S.S) EXAMINATION
Third Semester
Programme - M.Sc. Mathematics
PG3MATC15 - PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Time : Three Hours Maximum Weight : 30

Part A

Short Answer Questions

(Answer any **Eight** questions. Each question carries Weight 1)

1. Verify that $(x-a)^2+(y-b)^2+z^2=1$ is a complete integral of $z^2(1+p^2+q^2)=1$.
2. Find the domain in which the equations $xp-yq-x=0$ and $x^2p+q-xz=0$ are compatible.
3. State the heat conduction problem in an infinite rod.
4. Determine the partial differential equation satisfied by all surfaces of revolution with the z–axis as the axis of revolution.
5. Determine the characteristic curves of the equation $xz_y-yz_x=z$.
6. State the Neumann problem.
7. State the Hadamard's conditions for a well posed problem.
8. Define Volterra integral equation of second kind and give an example.
9. Show that the kernel $K(x,\xi)=(\sin x)(\cos\xi)$ has no characteristic numbers associated with $(0,2\pi)$
10. Define separable kernel and give an example.

$(8\times 1=8$ Weight$)$

Part B

Short Essay Questions

(Answer any **Six** questions. Each question carries Weight 2)

11. Find the general integral of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$.
12. Find the complete integral of $p+q-pq=0$.
13. Reduce the equation $u_{xx}+x^2u_{yy}=0$ to a canonical form.

14. Explain the terms 'domain of dependence' and 'range of influence'.

15. Show that the solution of the Dirichlet problem, if it exists, is unique.

16. State and prove the maximum principle for a harmonic function.

17. Determine the resolvent kernel associated with $K(x, \xi) = \cos(x+\xi)$ in $(0, 2\pi)$ in the form of a power series in λ.

18. Show that the characteristic functions of the Fredholm integral equation with symmetric kernel, corresponding to the distinct characteristic numbers are orthogonal over the interval (a, b).

(6 × 2 = 12 Weight)

Part C

Long Essay Questions

(Answer any **Two** questions. Each question carries Weight 5)

19. Show that the Pfaffian differential equation

$$\overrightarrow{X} \cdot d\overrightarrow{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$$

is integrable if and only if $(\overrightarrow{X} \cdot \text{curl } \overrightarrow{X}) = 0$.

20. Find the integral surface of the equation $x^3 p + y(3x^2 + y)q = z(2x^2 + y)$, which passes through the curve $x_0 = 1$, $y_0 = s$, $z_0 = s(1 + s)$

21. (a) Solve Neumann problem for the upper half plane:
$u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty,\ y > 0,$
$u_y(x, 0) = g(x), \quad -\infty < x < \infty,$
with the conditions that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$ and $\displaystyle\int_{-\infty}^{\infty} g(x)\, dx = 0.$
(b) Show that Cauchy problem of $u_{xx} + u_{yy} = 0$, with the initial data prescribed on the x axis are $u(x, 0) = 0$ and $u_y(x, 0) = \dfrac{\sin nx}{n}$ is not stable.

22. (a) Convert the differential equation $\dfrac{d^2y}{dx^2} + \lambda y = f(x)$, given $y(0) = 1$, $y'(0) = 0$ into integral equation.
(b) Solve $y(x) = x + \lambda \displaystyle\int_0^1 (x\xi^2 + x^2\xi)\, y(\xi)\, d\xi.$

(2 × 5 = 10 Weight)

SEMESTER IV

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
PG4MATC16 - MULTIVARITE CALCULUS AND INTEGRAL TRANSFORMS

(5 hours/week) **(Credit : 4)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO **No**	**Course Outcome**	**Cognitive Level**	**CO Mapped to PSO**
1	Learn conceptual variations while advancing from one variable to several variables in calculus.	U	1,6,7
2	Apply multivariable calculus tools in physics, economics, optimization, and understand the architecture of curves and surfaces in plane and space etc.	Ap	2,4,5,6
3	Realize importance of Stokes' theorems in other branches of mathematics.	An, Ap	1,3,4,7
4	Compute derivatives using the chain rule or total differentials.	E	2,4

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text 1: Tom APOSTOL, Mathematical Analysis, Second edition, Narosa Publishing House.

Text 2: WALTER RUDIN, Principles of Mathematical Analysis, Third edition- International Student Edition.

Module	**Contents**	**Content Mapped to CO No**	**Hours**
I	**Text 1** **Chapter 11 : Fourier Series and Fourier Integrals** 11.15 The Weirstrass approximation theorem 11.16 Other forms of Fourier series 11.17 The Fourier integral theorem 11.18 The exponential form of the Fourier integral theorem 11.19 Integral transforms 11.20 Convolutions 11.21 The convolution theorem for Fourier transforms.	1	20

II	**Text 1** **Chapter 12 : Multivariable Differential Calculus** 12.1 Introduction 12.2 The directional derivative 12.3 Directional derivatives and continuity 12.4 The total derivative 12.5 The total derivative expressed in terms of partial derivatives 12.6 An application of complex- valued functions 12.7 The matrix of a linear function 12.8 The Jacobian matrix 12.9 the chain rule 12.10 Matrix form of the chain rule	1,4	20
III	**Text 1** **Chapter 12 : Multivariable Differential Calculus** 12.11 The mean value theorem for differentiable Functions 12.12 A sufficient condition for differentiability 12.13 A sufficient condition for equality of mixed partial derivatives **Chapter 13 : Implicit functions and Extremum problems,.** 13.1 Introduction 13.2 Functions with non-zero Jacobian determinant 13.3 The inverse function theorem (without proof) 13.4 The implicit function theorem (without proof) 13.5 Extrema of real-valued functions of one Variable 13.6 Extrema of real- valued functions of several variables	1,2,4	25
IV	**Text 2** **Chapter 10 : Integration of Differential forms** 10.1 to 10.4 Integration 10.5 to 10.7 Primitive Mappings 10.8 Partitions of Unity 10.9 Change of Variables 10.10 to 10.25 Differential Forms 10.33 Stoke's Theorem (without proof)	2,3	25

References

[1] Limaye Balmohan Vishnu, Multivariate Analysis, Springer.

[2] Satish Shirali and Harikrishnan, Multivariable Analysis, Springe

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code (Pages 2) **Reg. No.**
. **Name :**

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Fourth Semester
Programme : M.Sc. Mathematics
PG4MATC16 : MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. State the exponential form of Fourier Integral Theorem.
2. State the inversion formula for Fourier Transforms.
3. Define convolution. Prove that $f * g = g * f$.
4. Show that the directional derivative need not exists even if all the partial derivative exists.
5. Define total derivative of a function. Show that the total derivative of a linear function is the function itself.
6. If $f = (f_1, f_2, ...f_m)$ is differentiable at $c \in R^n$, prove that $\|f'(c)(v)\| \leq M\|v\|$.
7. Verify that the mixed partial derivatives $D_{1,2}f(x, y)$ and $D_{2,1}f(x, y)$ are equal if $f(x, y) = log(x^2 + y^2)$.
8. If $f = u + iv$ is a complex-valued function with a derivative at a point z in C, then prove that $J_f(z) = |f'(z)|^2$.
9. Define a differential form of order k.
10. State Stoke's theorem. $(8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. State and prove Fourier Integral Theorem.
12. State and Prove Weierstrass Approximation Theorem.

Turn Over

13. Compute the gradient vector at those points in R^2 where it exists if $f(x,y) = x^2 * y^2 log(x^2 + y^2)$, if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

13. Compute the gradient vector at those points in R^2 where it exists if $f(x,y) = x^2 y^2 log(x^2 + y^2)$, if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

14. If f is differentiable at C with total derivative T_c, then prove that the directional derivative $f^{'}(c;u)$ exists for every u in R^n and $T_c(u) = f^{'}(c;u)$.

15. State and prove Mean Value Theorem.

16. Verify that $D_{1,2}f(x,y)$ = $D_{2,1}f(x,y)$ where $f(x,y) = \frac{log(x^2+y^2)}{x^2+y^2+xy}$.

17. For every $f \in C(I^k), L(f) = L'(f)$.

18. Suppose $\omega = \sum_I b_I(x)dx_I$ is the standard presentation of a $k - form\ \omega$ in an open set $E \subset R^n$. If $\omega = 0$ in E then $b_I(x) = 0$ for every increasing k-index I and for every $x \in E$.

($6 \times 2 = 12$ weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. State Convolution Theorem.

20. State and Prove Chain Rule.

21. Assume that one of the partial derivatives $D_1(f), D_2(f), ...D_n(f)$ exists at c and that the remaining $n-1$ partial derivatives exists in some $n-ball\ B(c)$ and are continuous at c. Prove that f is differentiable at c.

22. Suppose F is a $C^{'}$-mapping of an open set $E \in R^n$ into R^n,$0 \in E$, $F(0) = 0$, and $F^{'}(0)$ is invertible, then there is a neighborhood of 0 in R^n in which a representation
$F(x) = B_1..., B_{n-1}G_n o.....oG_1(x)$ is valid.

($2 \times 5 = 10$ weight)

ELECTIVES
GROUP A

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE01 – ALGORITHMIC GRAPH THEORY

(5 Hours/week) **(Credits : 3)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to :

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand the basics of different types of graphs.	U	4,5
2	Use effectively algorithmic techniques to study basic parameters and properties of graphs.	Ap	1,5
3	Design efficient algorithms for various optimization problems on graphs.	C	2,5,6
4	Model real world problems using graphs.	Ap	1,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : Gray Chartrand and O.R Oellermann, Applied and Algorithmic Graph Theory, Tata McGraw- Hill Companies Inc.

Module	Content	Content Mapped to CO	Hour
1	**Chapter 1 : Introduction to Graphs and Algorithms** 1.1 What is graph? 1.2 degree of a vertex 1.3 Isomorphic graphs 1.4 Subgraphs 1.5 Degree sequences 1.6 Connected graphs 1.7 Cut vertices and blocks 1.8 Special graphs 1.9 Digraphs 2.1 Algorithmic complexity 2.2 Search algorithms 2.3 Sorting algorithms 2.5 Greedy algorithms 2.6 Representing graphs in a computer	 1 1 1,4 1 1,4 1,2,4 1,2,4 1,2,3,4 4 2,3 2,3 2,3,4 3,4 3,4	(24 hrs)

2	**Chapter 3 : Trees, paths and distances** 3.1 Properties of trees 3.2 Rooted trees 3.3 Depth-first search 3.4 Breadth – first search 3.5 The minimum spanning tree problem 4.1 Distance in a graphs 4.2 Distance in weighted graphs 4.3 The centre and median of a graph 4.4 Activity digraphs and critical paths.	1 1,2 1,2,3,4 1,2,3,4 3,4 3,4 3,4 1,4 1,2	(22 hrs)
3	**Chapter 5 :Networks** 5.1 An introduction to networks 5.2 The max-flow min-cut theorem 5.3 The max-flow min-cut algorithm . 5.5 Connectivity and edge connectivity . Mengers theorem.	1 1,2,3,4 1,2,3,4 1,2,3,4	(22 hrs)
4	**Chapter 3 : Matchings and Factorizations** 6.1 An introduction to matchings 6.2 Maximum matchings in a bipartite graph 6.4 Factorizations 6.5 Block Designs	1 1,2 1,4 1,2,3,4	(22 hrs)

Reference

[1] Alan Gibbons, Algorithmic Graph Theory, Cambridge University Press, 1985

[2] Mchugh. J.A, Algorithmic Graph Theory, Prentice-Hall, 1990

[3] Golumbic. M, Algorithmic Graph Theory and Perfect Graphs, Academic press

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Fourth Semester
Programme : M.Sc. Mathematics
PG4MATE01- ALGORITHMIC GRAPH THEORY

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Construct a graph of order 5 whose vertices have degrees $1, 2, 2, 3, 4$. What is the size of this graph?
2. Write an algorithm to determine the first word alphabetically from a list of n words, and output this word and its location in the list.
3. What is adjacency matrix of a graph? Draw the graph G be with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E(D) = \{v_1v_2, v_1v_3, v_2v_3, v_3v_4, v_3v_5\}$. Find adjacency matrix of G.
4. Define a forest. Give an example.
5. State Cayleys Tree formula.
6. Define distance function on a graph G. Show that it is a metric.
7. Define vertex connectivity of a graph. Find $\kappa(K_{m,n})$.
8. Define an edge disjoint uv path in a graph G and the term $\lambda(u, v)$, where $u, v \in V(G)$.
9. Define a feasible vertex labeling of a weighted complete bipartite graph.
10. Define a $\{b, \nu \cdot r, k, \lambda\}$ design and state Fisher's inequality.

($8 \times 1 = 8$ weight)

Part B
Short Essay Questions

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Define (a) a non-separable graph, (b) a block, (c) an end-block in a graph. Give examples for each.

12. (a) Explain indegree, outdegree and degree of a vertex in a digraph. Draw a digraph and find indegree, outdegree and degree of each vertex.

    (b) State and prove The First Theorem on Digraph Theorey.

13. If T is a balanced complete binary tree of height h and order p, then prove that $h = \lceil \log_2 \left(\frac{p+1}{2}\right) \rceil$

14. Explain BFS Algorithm.

15. Define a flow in a network N. Give an example of a flow where flow along each arc is a positive integer.

16. In a network, show that the value of a maximum flow equals the capacity of a minimum cut.

17. Let G be a bipartite graph with partite sets V_1 and V_2. Prove that the set V_1 can be matched to a subset of V_2 if and only if V_1 is non deficient.

18. Prove that every bridgeless cubic graph contains a 1-factor.

$(6 \times 2 = 12$ weight)

Part C
Long Essay Questions

*(Answer any **two** questions. Each question carries Weight **5**)*

19. (a) An edge e of a connected graph is a bridge if and only if e does not lie on any of the cycle on G.

    (b) Show that every uv walk in a graph contains a uv path.

20. Write an algorithm to determine a critical path in an activity digraph D with start vertex S and terminal vertex T.

21. State and prove a necessary and sufficient condition that a flow f in a network N with underlying digraph D is a maximum flow.

22. State and prove Berge's theorem to determine the maximum matching in a graph G.

$(2 \times 5 = 10$ weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE02 - COMBINATORICS

(5 hours/week) **(Credit : 3)**

(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	**Course Outcome**	**Cognitive Level**	**CO Mapped to PSO**
1	Understand and explain concepts in combinatorics and group algebra, social network analysis, first-order logic and knowledge bases.	U, Ap	1,6,7
2	Create solutions to most of the counting problems in real life situation in a quick manner.	C	3,4,5
3	Analyze the problems which include the theory of permutation, combination, arrangements etc.	An	2,3,4
4	Apply theorems to solve the problems in set theory.	Ap	2,4,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: **Chen Chuan -Chong, Koh Khee Meng**, Principles and Techniques inCombinatorics, World Scientific, 1999.

Module	**Contents**	**Content Mapped to CO**	**Hours**
I	**Chapter 1 -Permutations and Combinations** 1.1 Two basic counting principles, 1.2 Permutations, 1.3 Circular permutations 1.4 Combinations 1.5 The injection and bijection principles 1.6 Arrangements and selection with repetitions 1.7 Distribution problems	1,3	30
II	**Chapter 3-The Pigeonhole Principle and Ramsey Numbers** 3.1 Introduction 3.2 The piegeonhole principle 3.3 More examples 3.4 Ramsey type problems and Ramsey numbers 3.5 Bounds for Ramsey numbers	1,2,4	15

III	**Chapter 4- Principle of Inclusion and Exclusion** 4.1 Introduction 4.2 The principle 4.3 A generalization 4.4 Integer solutions and shortest routes 4 . 5 Surjective mappings and Sterling numbers of the second kind 4.6 Derangements and a generalization 4.7 The Sieve of Eratosathenes and Euler φ-function.	1,3,4	20
IV	**Chapter 5- Generating Functions** 5.1 Ordinary generating functions 5.2 Some modelling problems, 5.3 Partitions of integer 5.4 Exponential generating functions **Chapter 6- Recurrence Relations** 6.1 Introduction 6.2 Two examples 6.3 Linear homogeneous recurrence relations, 6.4 General linear recurrence relations, 6.5 Two applications	1,2,4	25

References

[1] V Krishnamoorthy, Combinatorics theory and applications, E. Hoewood, 1986.

[2] Hall,Jr, Combinatorial Theory, Wiley- Interscinice, 1998.

[3] Brualdi, R A, Introductory Combinatorics, Prentice Hall,1992.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Fourth Semester
Programme : M.Sc. Mathematics
PG4MATE02 : COMBINATORICS

Time : Three Hours Maximum Weight : 30

Part A

Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. State injection and bijection Principle.
2. In how many ways can 5 boys and 3 girls be seated around a table if there is no restiction.
3. State the generalised Pigeonhole Principle.
4. State Ramsey's Theorem.
5. State the fundamental theorem of arithmetic.
6. Find the value of ϕ(100)
7. Define stirlings number of second kind.
8. Find the value of $D(4, 3, 1)$.
9. Write the r^{th} order linear recurrence relation for the sequence $\{a_r\}$.
10. Define Ordinary Generating functions for the sequence $\{a_r\}$.

($8 \times 1 = 8$ weight)

Part B

Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Show that $(4n)!$ is a multiple of $2^{3n}.3^n$, for each natural number n.
12. Show that if $|X| = n$, then $|P(X)| = 2^n$
13. Show that $R(3, 3) = 6$.

Turn Over

14. For all integers $p, q \geq 2$, $R(p, q) \geq R(p-1, q) + R(p, q-1)$.

15. Derive an expression for $D(n, r, k)$.

16. Let $S = \{1, 2, 3, ...500\}$. Find the number of integers in S which are divisible by 2, 3 or 5.

17. Solve the recurrence relation $a_n = a_{n-1} + a_{n-1}$.

18. Solve the recurrence relation $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3}$.

(6 $\times$ 2 = 12 weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. Define Stirling's number of first kind. Prove that $S(r, n) = S(r-1, n-1) + nS(r-1, n)$, where $r, n \in N$ with $r \geq n$.

20. Define Ramsey number, generalized Ramsey number. Show that $R(3, 3) = 6$.

21. State and prove generalized principle of inclusion and exclusion. Hence prove the Principle of inclusion and exclusion.

22. Let a_n denote the number of parallelogram of the n^{th} subdivision of triangle ABC.

(2 $\times$ 5 = 10 weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE03 - QUEUEING THEORY

(5 hours/week) **(Credit : 3)**
(Total Hours: 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO mapped to PSO
1	Construct models in discrete and continuous time based on Markov Chains,	C	1,4,6
2	Describe and explain the theory of Markov Chains, renewal process	U	2
3	Describe Little's formula and its applications	U	1,4,7
4	Describe and analyze basic Markov queuing models and situations to which they may be applied apply Markov models for selected applications	Ap, An	3,5,6,
5	To develop the modeling and mathematical skills to analytically determine computer systems and analytically determine computer systems and communication network performance	C	3,5,6
6	Understand the current performance analysis and queueing theory literature	U	4,5
7	Understand strengths and weaknesses of Queueing Models	U	5,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text book: Stochastic Models in Queueing Theory, Second edition, J. Medhi

Module	Content	Content Mapped to CO No	Hours
I	**Chapter 1: Stochastic processes** 1.1 Introduction 1.2 Markov chain 1.3 continuous time Markov Chain 1.4 Birth and Death Processes 1.5 Poisson Process 1.6 Randomization Derived Markov chain 1.7 Renewal processes 1.8 Regenerative processes 1.9 Markov renewal processes and Semi Markov processes	 2 1,2,5 4,7 1,5 5,6 1,2 2	25

II	**Chapter 2: Queueing systems- General Concepts** 2.1 Introduction 2.2 Queueing processes 2.3 Notations 2.4 Transient and steady state behaviour 2.5 Limitations of the steady state Distribution 2.6 Some general relationships in Queueing theory 2.7 Poisson arrival processes and its characteristics	 3,5,6 5,7 1,3,5 6,7 1,5,6 3,5	20
III	**Chapter 3: Birth and death Queueing systems: Exponential Models** 3.1 Introduction 3.2 The simple M/M/1 queue 3.3 The simple M/M/1 / k model 3.4 Birth and Death processes: exponential model, 3.5 M/M/∞models	 1,4 1,4,5 4,7 4,7	25
IV	**Chapter 3: Birth and death Queueing systems: Exponential Models** 3.6 The M/M/C model 3.7. The M/M/C/C system: The Erlang loss model 3.8. Model with finite input source. 3.9 transient Behaviour	 1,4,5 4,7 1,4 5,6,7	20

References

[1] Fundamentals of Queueing theory, Fourth Edition, Wiley, Donald Gross, John F Shortle, james M. Thompson, Carl M Harris.

[2] An Introduction to Queueing Theory: Modeling and Analysis in Applications, U Narayana Bhat

[3] Stochastic Models in Queueing Theory, Second Edition, J Medhi

[4] Gross D. and Harris C. M., Fundamentals of Queueing Theory, Wiley, 2012.

[5] Kleinrock L., Queueing Systems Volume 1 : Theory, Wiley, 2013 .

[6] Kleinrock L., Computer Applications, Volume 2, Queueing Systems, Wiley, 2013

QUESTION PAPER PATTERN

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER

M.Sc. DEGREE (C.S.S.) EXAMINATION

Fourth Semester

Programme - M.Sc. Mathematics

PG4MATE03– QUEUEING THEORY

Time: Three Hours Maximum Weight: 30

PART A

Short Answer Questions/Problems

(Answer any **eight** questions. Each question carries Weight **1**)

1. Define Markov chain and transition probability matrix.
2. What are the characteristics of Q-matrix?
3. The transition Probability matrix of a Markov Chain with three states 0,1,2 is given by $\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$ and the initial distribution is (0.6.0.3.0.1). Find the probability for $X_2 = 3$.
4. Define traffic intensity.
5. Write a note on residual life time and excess life time.
6. What is PASTA Property.
7. Derive an expression for expected number of customers in an M/M/1/K queueing system in unit time.
8. What effect does on doubling λ and μhave on L, L_q, W, W_q in an M/M/1 model.
9. Define an expression for expected number of busy servers in an M/M/C model.
10. Derive Erlang's First formula

(8x1=8 Weights)

PART B

Short Essay Questions/Problems

(Answer any **six** questions. Each question carries Weight **2**)

11. Let S= {0,1,2} be states of the Markov Chain with TPM $P = \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \\ 3/4 & 1/4 & 0 \end{bmatrix}$ Find its invariant Measure?
12. State and prove Chapman Kolmogorov Equation.
13. What are the basic characteristics of a queueing model?
14. State and prove Burke's theorem

15. Find the probability that a customer's wait in queue exceeds 20 min for an M/M/1/3 model with λ=4/hr and $1/_{\mu} = 15$ per minute.
16. Derive an expression for expected number of customers in the system for M/M/∞ model.
17. Derive an expression for expected umber of busy channels in M/M/c/c model.
18. Using difference equation technique derive transient state distribution for M/M/1 model.

(6x2=12 Weights)

PART C

Long Essay Questions/Problems

(Answer any two questions. Each question carries Weight 5)

19. Model M/M/1 queue as a pure birth death process
20. State and prove Little's formula
21. Show that the steady state distribution of n in the system in an M/M/∞ queue is Poisson.
22. Find the expected number in the system in an M/M/C queue

(2x5=10 Weights)

ELECTIVES
GROUP B

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards

FOURTH SEMESTER

(Elective) PG4MATE04 – ADVANCED COMPLEX ANALYSIS

(5 hours/week) **(Credit : 3)**

(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Explain continuous functions in the complex plane, Poles and singularities	U	1,6
2	Extend the domain over which a complex analytic function is defined	Ap	1,4
3	Factorize Complex functions using Weirestrass factorization theorem	E	24
4	Investigate simple connectedness using Rung's Theorem	An	2,3,4
5	Check the behavior of the radius of convergence for an analytic continuation along a curve.	E	3,6
6	Recite Jensn's Formula and Hadamard Factorization theorem and able to apply them to find genus and order of entire function	U, Ap	1,2,4,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : John B. Conway, Functions of One Complex Variable, Second Edition.

Module	Contents	Content Mapped to CO No	Hours
I	**Chapter VI1: Compactness and Convergence in the Space of Analytic Functions** 1 The space of continuous functions C(G, Ω) 2 Spaces of analytic functions 4 The Riemann Mapping Theorem (Statement Only) 5 Weirerstrass Factorization Theorem 6 Factorization of sine Function	 1 1 1,2 3 3	25 Hours

II	**Chapter VII : Compactness and Convergence in the Space of Analytic Functions** 7 The gamma function 8 The Riemann zeta Function **Chapter VIII : Runge's Theorem** 1 Runge's Theorem 3 Simple Connectedness 4 Mittag Leffler's Theorem	3 3 4 4 4	25 Hours
III	**Chapter IX: Analytic Continuation and Riemann Surfaces** 1 Schwarz Reflection Principle 2 Analytic Continuation along a path 3 Mondromy Theorem (without proof)	5 5 5	20 Hours
III	**Chapter XI: Entire Functions** 1 Jensen's Formula 2 The genus and order of an entire function 3 Hadamard Factorization Theorem (without proof)	6 6 6	20 Hours

References

[1] **H. Cartan**: Elementary Theory of analytic functions of one or several variables; Addison - Wesley Pub. Co.; 1973

[2] **B. Chaudhary**, The elements of Complex Analysis, Wiley Eastern

[3] **T.W. Gamelin**: Complex Analysis; Springer-Verlag, NY Inc.; 2001

[4] **S. Lang**, Complex Analysis, Springer

[5] **T.O. Moore and E.H. Hadlock**: Complex Analysis, Series in Pure Mathematics-Vol. 9; World Scientific; 1991

[6] **L. Pennisi**: Elements of Complex Variables (2nd Edn.); Holf, Rinehart & Winston 1976

[7] **H.A. Priestly**, Introduction to Complex Analysis, Clarendon press, Oxford, 1990

[8] **R. Remmert**: Theory of Complex Functions; UTM , Springer-Verlag, NY; 1991

[9] **W. Rudin**: Real and Complex Analysis (3rd Edn.); Mc Graw - Hill International Editions; 1987

[10] **H. Sliverman**: Complex Variables; Houghton Mifflin Co. Boston; 1975.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Fourth Semester
Programme - M.Sc. Mathematics
PG4MATE04 - ADVANCED COMPLEX ANALYSIS

Time: Three Hours Maximum Weight: 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. If f is a meromorphic function on an open set G, then prove that there are functions g and h on G such that $f = g/h$
2. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$
3. Define locally bounded set and also state Motel's theorem.
4. Let G be an open connected subset of C. If G is simply connected prove that $n(\gamma;a) = 0$ for every closed rectifiable curve γ in G and every point a in C –G.
5. State Schwarz Reflection principle.
6. State Monodromy theorem.
7. Define harmonic function and prove that if u : G → C is harmonic then u is infinitely differentiable.
8. State two versions of maximum principle.
9. State Poisson Jensen formula.
10. Define rank and genus of an entire function.

(8 x 1 = 8 Weights)

Part B
Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove that C(G, Ω) is a complete metric space.
12. State and prove Bohr – Mollerup theorem.
13. Prove that if $a \in C - K$, then $(z-a)^{-1} \in B(E)$.
14. Let $\gamma : [0, 1] \to \mathbb{C}$ be a path from a to b and let $\{(f_t, D_t) : 0 \le t \le 1\}$ be an analytic continuation γ . There is a number $\epsilon > 0$ such that if $\sigma : [0,1] \to C$ is any path from a to b with $|\gamma(t) - \sigma(t)| < \epsilon$ for all t , and if $\{(g_t, B_t) : 0 \le t \le 1\}$ is any continuation along σ with $[g_0]_a = [f_0]_a$ then prove that $[g_1]_b = [f_0]_b$.

Turn Over

15. Derive Harnack's inequality.
16. Prove maximum principle second version
17. Derive Jenson's formula
18. Let f be a non-constant entire function of order λ with $f(0) = 1$, and let $\{ a_1, a_2 \ldots \}$ be the zeros of f counted according to multiplicity and arranged so that $|a_1| \leq |a_2| \ldots$ If an integer $p > \lambda - 1$ then prove that $\frac{d^p}{dz^p}\left[\frac{f'(z)}{f(z)}\right] = -p\,!\,\sum_{n=1}^{\infty}\frac{1}{(a_n - z)^{p+1}}$ for $z \neq a_1, a_2, \ldots$

(6 x 2 = 12 Weights)

Part C

Long Essay Questions/Problems

(Answer any two questions. Each question carries Weight 5)

19. State and prove Arzela- Ascoli theorem.
20. State and prove Mittag – Leffler theorem
21. State and prove Harnack's principle
22. Let f be an entire function of genus μ. Then prove that for each positive number α there is a number r_0 such that for $|z| > r_0$, $|f(z)| < \exp(\alpha |z|^{\mu+1})$

(2x5=10 Weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) –PG4MATE05 - STOCHASTIC PROCESSES

(5 Hours/week) **(Credit 3)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO No
1	Understands Stochastic Process and classify the General Stochastic Process	U	1,6
2	Carry out derivations involving conditional probability distributions and conditional expectations	U	1,4
3	Define basic concepts from the theory of Markov chain, Renewal process and Queuing theory and prove the most important theorems	R	1,6
4	Compute probabilities of transition between states and return to the initial state after long time intervals in Markov chains	Ap	1,2,3
5	Determine limit probabilities in Markov chains after an infinitely long period	Ap	1,2,4,5
6	Derive differential equations for continuous time Markov processes with a discrete state space.	An. Ap	1,3,4,5
7	Solve differential equations for distributions and expectations in continuous time processes and determine corresponding limit distributions	Ap. E	1,2,3,5

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book : A First Course in Stochastic Processes, Second edition, Samuel Karlin, Howard M. Taylor, Academic Press New York San Francisco London

Module	Contents	Contents Mapped to CO	Hours
1	**Chapter 1. Elements of Stochastic Processes** 1. Review of Basic Terminology and Properties of Random Variables and Distribution functions, 2. Two Simple Examples of Stochastic Processes 3. Classification of General Stochastic Processes	 1,2 1,2 1,2	**20**
2	**Chapter 2. Markov Chains** 1. Definitions 2. Examples of Markov Chains 3. Transition Probability Matrices of a Markov Chain 4. Classification of States of Markov Chain 5. Recurrence 6. Examples of Recurrent Markov Chains	 3	**20**

3	**Chapter 3 The Basic Limit Theorem of Markov Chains and Applications** 1. Discrete Renewal Equation 3. Absorption Probabilities 4. Criteria for Recurrence 5. A Queueing Example 6. Another Queueing Model 7. Random Walk **Chapter 4 Classical Examples of Continuous Time Markov Chains** 1. General Pure Birth Processes and Poisson Processes 2. More about Poisson Processes 3. A counter Model	 3 3,4 3,4 3,4 3,4,5 3 3,6 3,6 3	**25**
4	**Chapter 4 Classical Examples of Continuous Time Markov Chains** 4. Birth and Death Process 5. Differential equations of Birth and Death Processes 6. Examples of Birth and Death Processes 7. Birth and Death Processes with Absorbing States 8. Finite State Continuous Time Markov Chains **Chapter 5 Renewal Processes** 1. Definition of a Renewal Process and Related Concepts 2. Some Examples of Renewal Processes	 6 6,7 3 3,5 1,3,4 3 3	**25**

References

[1] Stochastic Processes, Second Edition, Sheldon M Ross, Wiley series in Probability and Mathematical Statistics.

[2] Stochastic Processes Second Edition J. Medhi New age international Publishers.

[3] Stochastic processes in Physics and Chemistry, N G Van Kamp

Question Paper Pattern

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	Part A Short Answer Questions	Part B Short Essay Questions	Part C Long Essay Questions
Module I	3	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	3	2	1
Total	**10**	**8**	**4**

QP Code **(Pages 2)** **Reg. No.**

Name.

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Fourth Semester Programme - M.Sc. Mathematics
PG4MATE05– STOCHASTIC PROCESSES

Time: Three Hours Maximum Weight: 30

Part A

(Answer **Eight** Questions)

Each question carries 1 Weight

1. Let X be a non-negative random variable with cumulative distribution function$F(x)$ =Pr $\{X \leq x\}$. Show that $E(X) = \int_0^1 1 - F(x)dx$
2. Explain covarience Stationary Stochastic Processess
3. Define martingale.
4. If $i \leftrightarrow j$ and i is recurrent Show that j is recurrent.
5. Define transient state and recurrent state in a Markov Chain.
6. Write a note on inventory model.
7. State the postulates of Poisson process.
8. Explain strong ergodicity and weak ergodicity
9. Using an example describe Renewal Process.
10. If $F(x)$ is a distribution such that $F(0) = 0$ and $F(x) < 1$ for some $x > 0$. Show that $F(x)$ is an exponential Distribution if and only if $F(x + y) - F(y) = F(x)[1 - F(y)]$ for all $x, y \geq 0$

Part B

(Answer **any Six** Questions, Each question carries 2 weight)

11. The number of accidents occuring in a factory in a week is a random variable with mean μ and variance σ^2. The numbers of individuals injured in different accidents are independently distributed each with mean ν and variance ρ^2. Determine the number of individuals injured in a week
12. Let X be a non negative discrete random variable with possible values $0, 1, 2, ...$ Show that $$E(X) = \sum_{n=0}^{\infty} \text{Pr}\{X > n\} = \sum_{k=1}^{\infty} \text{Pr}\{X \geq k\}$$
13. Consider the random walk on the integers such that $P_{i,i+1} = p, P_{i,i-1} = q$ for all $i, 0 < p < 1, p + q = 1$ Determine $P^n_{0,0}$
14. If the one step transition probability matrix of a markov chainis $P =\| P_{ij} \|$, then $P^n_{ij} = \sum_{k=0}^{\infty} P^r_{ik} P^s_{kj}$ for any fixed pair of non negative integers r and s satisfying $r + s = n$, where we define
$P^0_{ij} = 1$ if $i = j$
$P^0_{ij} = 0$ if $i \neq j$

15. Let $j \in C$, an aperiodic recurrent class. Then for $i \in T$ prove that
$lim_{n\to\infty} P^n_{ij} = \pi_i(C).lim_{n\to\infty} P^n_{jj} = \pi_i(C)\pi_j$

16. Derive Kolmogorov Differential Equation

17. Find the mean time until absorption for a pure birth death process.

Part C

(Answer **ANY TWO** Questions, Each question carries 5 Weight)

18. State and Prove Abel lemma.

19. Let $X(t)$ be a pure birth continuous time Markov Chain.Assume that
Pr { an event happens in $(t, t + h)/X(t) =$ odd} $= \lambda_1 h + o(h)$
Pr { an event happens in $(t, t + h)/X(t) =$ even} $= \lambda_2 h + o(h)$, where $o(h)/h \to 0$as $h \to 0$
Take $X(0) = 0$
a)Find the Following probabilities
$P_1(t)$ =Pr{$X(t)$ =odd},
$P_2(t)$ =Pr{$X(t)$ =even}.
b) Determine $E[X(t)]$

20. Derive an expression for probability for m events occuring in a time t in the Poisson Process.

21. In Yule Process Derive an expression for $P_{N_n}(t)$ =Pr{$X(t) = n/X(0) = n$}

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE06 - FRACTAL GEOMETRY

(5 Hours/week) **(Credit 3)**
(Total 90 Hours) (Maximum Weight 30)

Course Outcome

Upon successful completion of the course student will be able to :

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand the fundamentals of Fractal Theory.	U	4,6
2	Analyze the different methods of dimension.	An	1,4,6
3	Be able to construct and analyse a wide range of fractals.	C, An	1,6
4	Be able to analyze different iterated function systems and self similar sets.	An	1,4,6
5	Understand the geometrical properties of fractals..	U	5,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text : Kenneth Falconer, Fractal Geometry Mathematical Foundation and Application, Third edition, Wiley, 2014

Module	Content	Content Mapped to CO No	Hour
I	**Mathematical background and Box-counting dimension** 1.1 Basic set theory 1.2 Functions and limits 1.3 Measures and mass distributions 2.1 Box-counting dimensions 2.2 Properties and problems of box –counting dimension	 1 1 1,3 2,5 2,3	(20hrs)
II	**Hausdorff and packing dimension and measures** 3.1 Hausdorff measure 3.2 Hausdorff dimension 3.3 Calculation of Hausdorff dimension-simple examples	 1 1,2 1,2,3,5	(25hrs)

III	**Iterated function systems – self-similar and self-affine sets and Dynamical systems** 9.1 Iterated functions systems 9.2 Dimensions of self similar sets 13.1 Repellers and iterated function systems 13.2 The logistic map	 4,5 4,5 3,4,5 3,5	(25hrs)
IV	**Iteration of complex functions – Julia sets and the Mandelbrot set** 14.1 General theory of Julia set 14.2 Quadratic functions-the Mandelbrot set 14.3 Julia sets of quadratic functions	 1 3 3,4,5	(20hrs)

References

[1] Falconer K.J, The Geometry of Fractal sets ,Cambridge University Press, Cambridge, 1986

[2] Barnsley M F, (1988), Fractals Every where, Academic press

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Fourth Semester
Programme : M.Sc. Mathematics
PG4MATE06 - FRACTAL GEOMETRY

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Define measure on R^n.
2. State any 2 properties of box counting dimension.
3. Define Hausdorff dimension.
4. State the countable stability of Hausdorff dimension.
5. Define the iterated function system.
6. Explain the chaos game .
7. Define chaos.
8. Define the Julia set $J(f)$.
9. Define the Mandelbrot set.
10. Find the Julia set of $f(z) = z^2$.

($8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems
*(Answer any **six** questions. Each question carries Weight **2**)*

11. Find the dimension of the middle third Cantor set.
12. Describe Lebesgue measure on R and R^n.
13. State and prove the fundamntal property of Hausdorff measures.

Turn Over

14. Prove that for every non-empty bounded $F \subset R$, $dim_H F \leq \underline{dim_H} F \leq \overline{dim_H} F$.

15. Find the dimension of modified von Koch curve.

16. Construct the tent map.

17. Let w be an attractive fixed point of f. Then prove that $\partial A(w) = J(f)$ and the same is true if $w = \infty$.

18. Prove that $J(f)$=$\{z \in C$: the family f^k is not normal at $z\}$.

($6 \times 2 = 12$ weight)

Part C
Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. Prove the equivalence of the following statement: (a) the smallest number of sets of diameter at most δ that cover F (b) the number of δ -mesh cubes that intersect F (c) the largest number of disjoint balls of radius δ with centres in F

20. (a) Prove the scaling property of Hausdorff measure. (b) Prove that every set $F \subset R^n$ with $dim_H F \subset 1$ is totally disconnected.

21. Explain the dynamics of logistic map.

22. Prove that for $c \in C$, the Julia set $J(f_c)$ is connected if the sequence of iterates $\{f_c^k(0)\}_{k=1}^{\infty}$ is bounded and is totally disconnected otherwise.

($2 \times 5 = 10$ weight)

ELECTIVES
GROUP C

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE07 - DIFFERENTIAL GEOMETRY

(5 hours/week) **(Credit : 3)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Compute quantities of geometric interest such as curvature, tangent space etc.	E	2,4
2	Understand the idea of orientable/non-orientable surfaces.	U	1,2,6
3	Analyze the concept of a parameterized surface with the help of examples.	An	3,6
4	Evaluate the length of curves.	E, Ap	2,4,7

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: John A. Thorpe, Elementary Topics in Differential Geometry

Module	Contents	Content mapped to CO	Hours
I	Chapter 1 - Graphs and level sets Chapter 2 - Vector Fields Chapter 3 - The Tangent Space Chapter 4 – Surfaces Chapter 5 - Vector fields on surfaces, Orientation.	1, 3	15
II	Chapter 6 – The Gauss Map Chapter 7 - Geodesics Chapter 8 - Parallel Transport	1	20

III	Chapter 9 - The Weingarten Map Chapter 10 – Curvature of Plane Curves Chapter 11 – Arc Length and Line Integrals	1,3,4	25
IV	Chapter 12 - Curvature of Surfaces Chapter 13 – Parametrized Surfaces Chapter 15 - Local Equivalence of Surfaces and Parametrized Surfaces	2,3	30

References

[1] Serge Lang, Differential Manifolds.

[2] I.M. Siger, J.A Thorpe, Lecture notes on Elementary topology and Geometry, Springer – Verlag, 1967.

[3] S. Sternberg, Lectures on Differential Geometry, Prentice-Hall, 1964.

[4] M. DoCarmo, Differential Geometry of curves and surface.

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code. (Pages 2) **Reg. No.**

Name :

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S) EXAMINATION
Fourth Semester
Programme : M.Sc. Mathematics
PG4MATE07 - Differential Geometry

Time : Three Hours Maximum Weight : 30

Part A
Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Sketch the level curves of the function $f(x_1, x_2) = {x_1}^2 + {x_2}^2$.
2. Show that the n unit sphere ${x_1}^2 + {x_2}^2 + ... + {x_n}^2 = 1$ is connected if $n > 1$.
3. Show that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.
4. Find the velocity, acceleration and speed of the paramaterized curve $\alpha(t) = (cost, sint, t)$.
5. Show that a parametrized curve $\alpha : I \to S$ is a geodesic on a surface S if and only if its covariant acceleration is zero along α.
6. State the properties of Levi- Civita parallelism.
7. Find the curvature of the oriented plane curve $ax_1 + bx_2 = c, (a, b) \neq (0, 0)$.
8. Find the length of the parametrized $\alpha : [-1, 1] \to R^3$ by $\alpha(t) = (cos3t, sin3t, 4t)$.
9. Describe parametrized torus in R^3.
10. Define Gauss- Kronecker curvature of an n- surface in R^{n+1}.

$(8 \times 1 = 8$ weight)

Part B
Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight **2**)*

11. Prove that the set of all vectors tangent to $f^{-1}(c)$ at a regular point is contained in the $n-$dimensional vector subspace $[\nabla f(p)]^{\perp}$ of ${R_p}^{n+1}$.

Turn Over

12. Suppose $S \in R^{n+1}$ is an $n-$surface, $\alpha : I \to S$ be a parametrized curve in S. Prove that there exists a unique vectorfield V tangent to S along α which is parallel and has $V(t_0) = v$, for $t_0 \in I$ and $v \in S_{\alpha(t_0)}$.

13. Show that if $\alpha : I \to R^{n+1}$ is a parametrized curve with constant speed then $\dot{\alpha}(t) \perp \ddot{\alpha}(t)$ for all $t \in I$.

14. Prove the existence and uniqueness of a tangential vector field on an n-surface in R^{n+1}.

15. Show that weingarten map is self adjoint.

16. Compute the line integral $\int(-x_2dx_1 + x_1dx_2)$ over the ellipse $\frac{{x_1}^2}{a^2} + \frac{{x_2}^2}{b^2} = 1$.

17. Find the extreme values of the curve $g(x_1, x_2) = 2{x_1}^2 + 2x_1x_2 + {x_2}^2$.

18. Find the Gaussian Curvature of the parametrized 2- surface $\phi(t, \theta) = (cos\theta, sin\theta, t)$.

(6 × 2 = 12 weight)

Part C

Long Essay Questions/Problems

*(Answer any **two** questions. Each question carries Weight **5**)*

19. Prove the existence and uniqueness of a maximal integral curve through $p \in U \subset R_{n+1}$ in a smooth vector field X.

20. Prove that there exist a global parametrization of an oriented plane curve C if and only if C is connected.

21. Prove that in a compact oriented $n-$surface $S \in R^{n+1}$, there exist a point p such that the second fundamental form at p is definite .

22. Show that Gauss Map is a diffeomorphism in a compact connected oriented $n-$ surface having zero Gauss - Kronecker curvature.

(2 × 5 = 10 weight)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards
FOURTH SEMESTER
(Elective) PG4MATE08 THEORY OF WAVELETS

(5 hours/week) **(Credit : 3)**
(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO Mapped to PSO
1	Understand the fundamentals of Wavelet and the emerging fields of applications of Linear Algebra, Fourier Transforms and Hilbert Space.	U	1,4,6
2	Analyse problems and conduct researches related to theoretical and applied problems related to wavelet theory, and, more generally, time-frequency analysis	An	1,2,3,4
3	Participate in scientific discussions as well as to collaborate in joint interdisciplinary researches.	C	1,2,3,7
4	Apply mathematical techniques connected with signal and image processing, data banks to solve real life problems	E, Ap	2,3,4,5

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book:- Michael W. Frazier, An introduction to Wavelets through Linear Algebra, Springer- verlag, 2000.

Pre-requisites :- Linear Algebra, Discrete Fourier Transforms, Elementary Hilbert Space theorem. (No questions shall be asked from these sections.)

Module	Content	Content Mapped to CO	Hours
I	**Chapter 3: Wavelets on Z_N** 3.1.Construction of Wavelets on Z_N The First Stage.	1,4	(20 hours)
II	**Chapter 3: Wavelets on Z_N** 3.2. Construction of Wavelets on Z_N : The Iteration Step 3.3. Examples and application	1,2,4	(20 hours)

III	**Chapter 4: Wavelets on Z** 4.1. $l^2(Z)$ 4.2.Complete Orthonormal sets in Hilbert Spaces, 4.3. $L^2[-\pi, \pi]$ and Fourier Series.	1,2,3,4	(20 hours)
IV	**Chapter 4: Wavelets on Z** 4.4. The Fourier Transform and Convolution on $l^2(Z)$ 4.5. First-stage Wavelets on Z, 4.6. The Iteration step for Wave lets on Z 4.7. Implementation and Examples	1,2,3,4	(30 hours)

References

[1] Charles K. Chui, An Introduction to Wavelets, Academic (1992).

[2] Ingrid Daubechies, Ten Lectures on Wavelets, SIAM, (1992).

[3] K.R Unni, Wavelets, Frames and Wavelet Bases in L P Lecture notes, Bhopal (1997).

[4] Stephane Mallat, A Wavelet Tour Of Signal Processing, Academic Press (1999).

[5] Don Hong, Jianzhong Wang, Robert Gardner, Real Analysis with an Introduction to Wavelets, Elsevier Academic Press (2005).

[6] Yves Meyer, Wavelets and Operators, Cambridge University Press (1992).

[7] John. J Beneditto, Michael W. Frazier Wavelets-Mathematics and Applications, CRC, (1994). MATHEMATICS

[8] Eugenio Hernandez, Guido L. Weiss, First course on wavelets, CRC, (1996)

Question Paper Pattern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	3	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	3	2	1
Total	**10**	**8**	**4**

QP Code.

Reg No.

Name

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Fourth Semester
Programme - M.Sc. Mathematics
(ELECTIVE) PG4MATE08 – THEORY OF WAVELETS

Time: Three Hours Maximum Weight: 30

PART A

Short Answer Questions/Problems

*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Suppose $N = 2M, w^*(n) = (-1)^n w(n)$ in $l^2(\mathbb{Z}_N)$. Show that
$$\widehat{(w^*)}(m) = \hat{w}\left(m + \frac{N}{2}\right).$$
2. Write the vectors in Frequency domain that generates the first stage real Shannon basis.
3. Let $u = l^2(\mathbb{Z}_4)$ be such that $\hat{u} = (\sqrt{2}, \sqrt{2}, 0,0)$. Find some $\hat{v}$ such that $\{v, R_2 v, u, R_2 u\}$ is an orthonormal basis for $l^2(\mathbb{Z}_4)$.
4. Suppose N is divisible by 2, define $u_l \in l^2(\mathbb{Z}_{N/2^{l-1}})$ by $u_2(n) = u_1(n) + u_1(n + \frac{N}{2})$, prove that $\widehat{u_2}(m) = \widehat{u_1}(2m)$
5. Suppose $1 \leq p \leq n$, prove that $(n-1)2^{n-1} + (n-2)2^{n-2} + \cdots + (n-p)2^{n-1} \leq n2^n$.
6. Suppose H is Hilbert space and $\{a_j\}_{j\in\mathbb{Z}}$ is an orthonormal set in H. Prove that $\{a_j\}_{j\in\mathbb{Z}}$ is a complete orthonormal set if and only if $f = \sum_{j\in\mathbb{Z}}\langle f, a_j\rangle\, a_j$, for all $f \in H$.
7. Show that the trigonometric system is an orthonormal set in $L^2([-\pi, \pi))$.
8. Give an example of an absolutely integrable function which is not square integrable on $[-\pi, \pi)$.
9. Show that the system $\left\{e^{\frac{in\pi t}{a}}\right\}_{n\in\mathbb{Z}}$ is a complete orthonormal set in $L^2\big([-a, a)\big)$.
10. Describe homogenous wavelet system for $l^2(\mathbb{Z})$. (8 x 1 = 8 Weights)

Part B

Short Essay Questions/Problems

*(Answer any **six** questions. Each question carries Weight 2.)*

11. Suppose $z, w \in l^2(\mathbb{Z}_N)$, prove that $z * w(k) = \langle z, R_k\widetilde{w}\rangle$, for any $k \in \mathbb{Z}$.
12. Let $z, w, u, v \in l^2(\mathbb{Z}_N)$, prove that $\langle R_k z, R_j w\rangle = \langle z, R_{j-k} w\rangle = \langle R_{k-j} z, w\rangle$.
13. Explain the construction of Daubechies's D_6 wavelets on $l^2(\mathbb{Z}_N)$.

14. Suppose $f \in L^1[-\pi,\pi)$ and $\langle f, e^{in\theta}\rangle = 0$ for all $n \in Z$, prove that $f(\theta) = 0$ a.e.
15. Suppose N is divisible by 2^l. Define $u_l \in l^2(Z_{N/2^{l-1}})$ be $u_1(n) = \sum_{k=0}^{2^{l-1}-1}\left(u_1(n + \frac{KN}{2^{l-1}})\right)$. Prove that $\widehat{u_1}(m) = \widehat{u_1}(2^{l-1}m)$.
16. Suppose l is a positive integer, $g_{l-1} \in l^2(\mathbb{Z})$ and $\{R_{2^{l-1}k}g_{l-1}\}_{k\in\mathbb{Z}}$ is orthonormal in $l^2(\mathbb{Z})$, the system matrix $A(\theta)$ of $u, v \in l^1(\mathbb{Z})$ is unitary for all θ. Define $f_1 = g_{l-1} * U^{l-1}(v)$ and $g_1 = g_{l-1} * U^{l-1}(u)$. Prove that $\{R_{2^l k}f_l\} \cup \{R_{2^l k}g_l\}_{k\in\mathbb{Z}}$ is orthonormal.
17. Suppose $M \in \mathbb{N}, N = 2M$ and $u, v \in l^2(Z_N)$ then show that $\{R_{2k}u\}_{k=0}^{M-1} \cup \{R_{2k}v\}_{k=0}^{M-1}$ is an orthonormal basis for $l^2(\mathbb{Z}_N)$ if and only if the system matrix $A(n)$ of u and v is unitary for each $n = 0,1,\ldots\ldots, M-1$
18. Suppose $N = 2^n, 1 \le p \le n$ and $u_{1,}v_{1,}u_{2,}v_{2,}\ldots\ldots., u_{p,}v_p$ form a p[th]-stage wavelet filter sequence. If $z \in l^2(\mathbb{Z}_N)$ show that the output $\{x_1, x_2, x_3 \ldots\ldots. x_p, y_p\}$ of the analysis phase of the corresponding p[th]-stage wavelet filter bank can be computed using not more than $4N + N\log_2 N$ complex multiplications.

(6×2=12 Weights)

Part C

Long Essay questions/Problems

*(Answer any **two** questions. Each question carries1 5 Weight.)*

19. Suppose $u \in l^1(\mathbb{Z})$ and $\{R_{2k}u\}_{k\in\mathbb{Z}}$ is orthonormal in $l^2(\mathbb{Z})$. Prove that if a sequence $v \in l^2(\mathbb{Z})$ defined by $v(k) = (-1)^{k-1}\overline{u(1-k)}$, then the system matrix $A(\theta)$ of u and v is unitary.
20. State and prove Plancherel's formula for $L^2([-\pi,\pi))$.
21. Show that the set of functions $\{1\} \cup \{\sqrt{2}\cos n\theta\}_{n=1}^{\infty} \cup \{\sqrt{2}\sin n\theta\}_{n=1}^{\infty}$ is a complete orthonormal system.
22. Suppose $z \in l^2(\mathbb{Z}_N)$. If N is even prove that $(\widehat{D(z)(n)} = \frac{1}{2}\left(\hat{z}(n) + \hat{z}\left(n + \frac{N}{2}\right)\right)$, where D is the downsampling operator.

(2×5=10 Weights)

M.Sc DEGREE PROGRAMME - 2022 Admission Onwards

FOURTH SEMESTER

(Elective) PG4MATE09- ORDINARY DIFFERENTIAL EQUATION

(5 hours/week) **(Credit : 3)**

(Total Hours : 90) (Maximum. Weight: 30)

Course Outcome:

Upon the successful completion of the course students will be able to:

CO No	Course Outcome	Cognitive Level	CO mapped to PSO
1	Understand the genesis of ordinary differential equations and classify the differential equations with respect to their order and linearity, check the independence using Wronskian, understand ordinary point singular point, Legendre Polynomial, Bessel Equation, Green's function	U	1,4
2	Apply the existence uniqueness theorem of differential equations, power series method, Picards'd theorem	Ap	4,5,7
3	Find solution of higher-order linear differential equations; solve systems of linear differential equations	E	1,2
4	Analyze real-world scenarios to recognize when ordinary differential equations (ODEs) or systems of ODEs are appropriate	An	1,3,5,7
5	Formulate problems about the scenarios creatively model these scenarios in order to solve the problems using multiple approaches	C	1,2,3,4,5
6	Judge if the results are reasonable, and then interpret and clearly communicate the results	E	1,6

R - Remember; U - Understanding; Ap - Apply; An - Analyse; E - Evaluate; C - Create

Text Book: Ordinary Differential Equations and Stability theory: by S G Deo and V Raghavendra.TMH-1980.

Prerequisites; Chapter 1upto Section 1.5. No questions shall be asked from this section

Module	Content	Content Mapped to CO No	Hours
I	**Chapter 1:Basic Concepts and Linear Equations of the First order:** 1.6. Classification 1.7. Initial and boundary value problems **Chapter 2: Linear Differential Equations of Higher order:** 2.1. Introduction 2.2. Linear dependence and Wronskian 2.3 Basic theory for linear equations 2.4. Method of variation of parameters 2.5. Two useful formulae 2.6 Homogeneous linear Equation with constant coefficients	1,3 1,5 1,2 1,5,6 1,3,5 1,3,5 1,5,6 2,3,4	**(20hrs)**
II	**Chapter 3: Solutions in Power series:** 3.1. Introduction 3.2. Second Order Linear Equations with Ordinary Points **3.**3. Legendre equation and Legendre polynomials 3.4. Second order equations with regular singular points **3.**5. Bessel equation	1,2 1,2,3 1,2,3,5 1,4,5,6 1,4,6	**(22 hrs)**
III	**Chapter 4: Systems of Linear Differential Equations.:** **4.1. Introduction** **4.2.** Systems of first order equations **4.**3. Existence and uniqueness theorem **4.**4. Fundamental matrices **4.**5. Non homogeneous linear systems **4.**6. Linear systems with constant co-efficient **4.**7. Linear systems with periodic co-efficient	1,3,4 2,3,5 1,3 1,3,4 1,3,4 1,3,4	**(24 hrs)**
IV	**Chapter 5: Existence and uniqueness of solutions.:** **5.1. Introduction** **5.2.** Preliminaries **5.**3. Successive approximation **5.**4. Picard's theorem **5.**5 Non uniqueness of solutions	2,3 1,4,5 1,2,6	**(24 hrs)**

	5.6. Continuation and dependence on initial conditions 5.7. Existence of solutions in the large **5**.8. Existence and uniqueness of solutions of systems **Chapter 7: Boundary Value Problems** 7.1. Introduction 7.2. Strum -Liouville's problem 7.3. Green's functions	2,3,4 2,5,6 1,2,6 3,4,5 1,5,6	

References

[1] Introduction to Ordinary Differential Equations: by E A Coddington. PHI-1961

[2] Coddington, E. and Levinson, N., Theory of Ordinary Differential Equations. McGraw-Hill, New York, 1955.

[3] Lawrence Perko, Differential equations and dynamical systems, Springer, 3rd Edition, 2001.

[4] G.F. Simmons: Differential Equations with Applications and Historical notes. Tata McGraw Hill, 2nd Edition, 2003.

[5] A. K. Nandakumaran, P. S. Datti and Raju K. George, Ordinary Differential Equations: Principles and Applications (Cambridge IISc Series), IISc Press, 2017.

[6] Hartman, Ordinary Differential Equations, Birkhaeuser, 1982.

Question paper patern

	Part A Weight 1	Part B Weight 2	Part C Weight 5
	Short Answer Questions	Short Essay Questions	Long Essay Questions
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

QP Code. **Reg No.**
Name

MODEL QUESTION PAPER
M.Sc. DEGREE (C.S.S.) EXAMINATION
Fourth Semester
Programme - M.Sc. Mathematics
(ELECTIVE) PG4MATE09 - ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum Weight: 30

PART A
Short Answer Questions/Problems
*(Answer any **eight** questions. Each question carries Weight **1**)*

1. Define the order and degree of a differential equation with examples
2. Solve the equation$(1 - e^{x}\ )\ \ dx/dt = 3t^{2}$
3. Show that sin$(x, sin2x, sin3x)$ are linearly independent on $[0.2\pi]$
4. Find the first three terms of the solution of $y' = t^{2} - y^{2}, y = 0\ for\ t = 0$ using power series method.
5. Locate and classify the singular points of the differential equation
$$(x^{2} - 3x)\ y'' + (x + 2)\ y' + y = 0$$
6. Prove or disprove : If Φ is a fundamental matrix for $x' = A(t)x$ and C is any constant non-singular matrix then $C\Phi$ is a fundamental matrix
7. Represent the linear equation $x''' - 6x'' + 11x' - 6x = 0$ in matrix form
8. Find the fundamental matrix for the system $x' = Ax$ where $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$
9. State Sturm Liouville theorem
10. Compute the first three successive approximations for the solutions of $x'' = e^{x}, x(0) = 0$

(8x1=8 Weight)

PART B
Short Essay Questions/Problems
*(Answer any **six** questions. Each question carries Weight **2**)*

11. Use the method of variation of parameters to find the general solution of $x''' - x' = cost.$
12. Find the three linearly independent solutions of the equation $x''' = 4x'' + 5x' - 2x = 0$ and hence find the solution $x(t)$ which satisfies $x(0) = 0, x'(0) = 0, x''(0) = 1$

Turn Over

13. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials then prove that

$$\int_{-1}^{1} P_m(t)P_n(t)dt = 0 \; if \; m \neq n$$

14. Prove that $t^{1/2}J_{1/2}(t) = \frac{\sqrt{2}}{\sqrt{\pi}}\sin(t)$.

15. Find the fundamental matrix for the system $x' = Ax$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}$

16. Find the eigen values and eigen functions of the equation
$x'' + \lambda = 0, 0 \leq t \leq \pi, x(0) = 0, x(\pi) = 0$

17. Check whether the following functions satisfy Lipschitz condition on the set S
a)$f(x,y) = xy^2; S: |x| \leq 1, |y| \leq \infty$

b) $f(x,y) = y^{2/3}; S: |x| \leq 1, |y| \leq 1$

18. If the Wronskian of two functions$x_1 \, and \; x_2$ in I is non zero for atleast one point of the interval I then prove that the function is non zero for atleast one point of the interval I

(6x2=12 Weight)

PART C

Long Essay Questions/Problems

(*Answer any two questions. Each question carries Weight 5)*

19. State and prove Picard's Theorem
20. Prove that the set of all solutions of the system $x' = A(t)x$ on I forms an n-dimensional vector space over the field of Complex numbers
21. $a_0(t), a_1(t), a_2\,(t), \ldots.. a_n(t), b(t)$ are real valued continuous functions of t defined on an interval I of the real line and $a_0(t) \neq 0, t_0 \in I$ Then prove that the initial value problem

$$a_0(t)x^{(n)} + a_1(t)x^{(n-1)} + a_2\,(t)x^{(n-2)} + \cdots..+a_n(t)x = \; b(t),$$
$$x(t_0) = a_1\,, x'(t_0) = a_2, \ldots\ldots x^{(n-1)}(t_0) = a_n$$

admits on and only one solution.

22. Let a_1, $a_2, \ldots$ be positive zeros of the Bessel function $J_p(t)$ then prove that$\int_0^1 t\,J_p(a_m t)J_p(a_n t)dt = 0 \; if \; m \neq n \; and$

$$\int_0^1 t\,J_p(a_m t)J_p(a_n t)dt = \frac{1}{2}\,J_{p+1}a_n^2 \; if \; m = n$$

(2x5=10 Weight)

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